Simultaneous Quasi-Phase Matching of Two Arbitrary Four-Wave-Mixing Processes

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Abstract—Efficient nonlinear four-wave-mixing (FWM) interactions can be realized by either phase-matching or quasi-phase-matching (QPM) the optical waves involved. Conventionally, this is only achieved for a limited set of FWM processes. In this paper, we develop a method to simultaneously realize QPM of two arbitrary FWM processes. This double QPM method combines two QPM techniques, namely QPM by phase-mismatch switching for one process, and QPM by dispersion compensation for the other. To our knowledge, this scheme is the first one that can realize efficient interactions for two arbitrary FWM processes in essentially any type of waveguide medium that is characterized by a single zero-dispersion point in the wavelength domain of interest. We apply the proposed scheme to design two devices that can be operated at larger pump-signal frequency differences than previously possible, namely a wavelength converter switch and a one-to-two Raman wavelength converter. Hence, the double-QPM method allows optical waves to interact through a larger number of nonlinear optical processes, and enables novel FWM-based applications.

Index Terms—Four-wave mixing, nonlinear optics, optical phase matching, optical wavelength conversion, quasi-phase-matching.

I. INTRODUCTION

Nonlinear four-wave-mixing (FWM) interactions enable a wide variety of photonic devices. By mixing waves at different frequencies, FWM enables functionalities such as wavelength conversion [1], [2], parametric gain [3], all-optical switching [4], and signal regeneration [5].

Essential for achieving efficient FWM interactions is the phase-matching condition [6]. Only those FWM processes that are phase-matched through dispersion-engineering [7], [8] or for which a quasi-phase-matching (QPM) scheme [9]–[13] is realized, result in an efficient nonlinear interaction. In practice, this condition is typically only satisfied for a limited set of FWM interactions. For instance, phase-matching through dispersion engineering requires the waves involved in the FWM process to be located symmetrically around a frequency near the zero-dispersion wavelength of the waveguide medium [7]. In this manner, efficient broadband FWM can be achieved [14], but only for a limited range of pump wavelengths tuned closely to the zero-dispersion wavelength. Alternatively, QPM schemes are based on varying the waveguide characteristics along the propagation path to enhance a FWM process [9]. However, since the variations are engineered in function of a given FWM process, the QPM enhancement remains specific to that process. Other FWM processes remain inefficient and have a negligible impact on the light propagation. Hence, the need to satisfy the phase-matching condition limits the number of efficient nonlinear interactions possible between a set of waves, and thus also the functionalities that can be achieved.

Recently, several schemes have been proposed to counter the conventional limits imposed by the phase-matching condition and enable efficient interactions for a larger number of FWM processes in a single waveguide. In the context of nonlinear optical fibers for instance, one-to-two [2], one-to-four [4], and even one-to-60 [15] wavelength converters based on non-degenerate, double-pump FWM schemes have been demonstrated. These schemes however require the signal frequency to be closely situated to one of the pump frequencies, and this limits the range of FWM processes for which a high efficiency can be obtained in the same device. Alternatively, by employing specially engineered optical fibers, more versatile phase-matching schemes can be achieved even in the presence of a single pump. First, by dispersion-engineering highly nonlinear fibers so that they exhibit not only zero dispersion near the pump wavelength of interest, but also a zero dispersion slope, phase-matching of a single-pump scheme can be achieved for a range of pump wavelengths [16]. The bandwidth attainable in these devices remains limited, as a conversion bandwidth of e.g., 35 nm can typically only be maintained over a pump wavelength range of about 30 nm. Second, by engineering photonic crystal fibers, up to three zero-dispersion points can be created to enable rich phase-matching topologies [17]. However, the flexibility of this approach remains unclear as the impact of such a topology was only investigated in the context of nonlinear propagation of ultrashort pulses. In the context of integrated waveguides, several other approaches have been described allowing phase-matching for multiple FWM configurations. First, countering the non-zero second-order dispersion by tailoring the higher-order dispersion results in a discrete conversion band for large pump-signal frequency detunings, while still providing the continuous conversion band for small detunings related with conventional phase-matching by dispersion-engineering [18]. With this approach the signals that are converted remain however inherently restricted to distant frequency bands. Second, QPM schemes realized for a large but fixed pump-signal frequency detuning often also provide a continuous conversion band for smaller pump-signal detunings which originates from conventional phase-matching [9], [12]. Since the latter conversion band...
is typically very small, these schemes offer little freedom for combining different FWM configurations. Hence, while all these schemes do offer more versatile FWM interactions, the FWM processes that can be combined in a single device remain very limited. The goal of this paper is to develop a novel scheme that can combine two FWM processes while allowing larger pump-signal frequency differences, as well as pump frequencies for the two processes that lie farther away from each other than is possible with the conventional approaches previously described. Moreover, we develop this scheme for commonly available waveguide structures that are characterized by a single zero-dispersion point in the wavelength domain of interest, without the need for any other necessary properties, such as a modifiable FWM gain as required for some QPM techniques [12], [19], [20]. In this paper, we focus on silicon-on-insulator (SOI) nanowire strip waveguides, and describe how our novel scheme can be implemented in such waveguide structures. In Section II, we start by reviewing the techniques possible for obtaining efficient interactions for a single FWM process in such waveguide structures, namely phase-matching through dispersion engineering, QPM by phase-mismatch switching (PMS) [9], and QPM by dispersion compensation [11], [13]. In Section III, we discuss whether these techniques can also be used to realize efficient interactions between two arbitrary FWM processes simultaneously in the waveguide media of interest. To do so, we consider the specific example of a SOI nano-scale strip waveguide in the near-infrared wavelength domain. We show that only a double-QPM scheme, in which one FWM process is PMS and the other by dispersion compensation, is compatible with the corresponding dispersion characteristics containing a single zero-dispersion point. We also outline a method for designing waveguides realizing such double QPM. To show the potential of the scheme proposed, we discuss in Section IV two devices that due to double QPM can be operated at larger pump-signal frequency differences than previously possible, namely a wavelength conversion switch and a one-to-two, single-pump Raman wavelength converter. Finally, the conclusions are presented in Section V. The mathematical model describing the one-to-two Raman wavelength converter is also briefly discussed in the appendix.

II. THEORY OF FWM

In this paper, we focus on the nonlinear process of degenerate FWM. This process describes the interaction of two pump photons with one signal and one idler photon, the frequencies of which are located symmetrically around the pump frequency, i.e., \( \omega_i = 2\omega_p - \omega_s \). Such FWM allows one to generate an idler wave out of a strong pump and weak signal input wave. An essential parameter for the degenerated FWM process is the phase mismatch \( \Delta \beta \) between the interacting waves [8], [21]:

\[
\Delta \beta = \Delta \beta_0 + \Delta \beta_{NL}.
\] (1)

Here \( \Delta \beta_0 = 2\beta_{0,p} - \beta_{0,s} - \beta_{0,i} \) is the difference between the pump, signal and idler propagation constants, and \( \Delta \beta_{NL} \) the phase mismatch induced by the optical nonlinearity. Since our discussion revolves around the phase-mismatches related to different degenerate FWM processes, it should be noted that all results obtained can simply be transposed to non-degenerate FWM or any other nonlinear mixing process by substituting the appropriate phase-mismatch formulas.

The phase mismatch \( \Delta \beta \) is essential for the degenerate FWM process because it determines how the FWM gain evolves along the optical propagation path. For large phase mismatches, the sign of the nonlinear gain oscillates along the propagation path, effectively disrupting a strong power build-up. To achieve efficient FWM interactions the waves either have to be phase-matched, or a QPM scheme has to be realized.

Conventionally, a positive FWM gain is maintained across the whole propagation length by phase-matching the interacting waves, i.e., by employing a small phase mismatch \( \Delta \beta \approx 0 \) [7], [8]. To do so, the phase-mismatch due to the linear dispersion \( \Delta \beta_0 \) should compensate the one induced by the optical nonlinearity, \( \Delta \beta_{NL} \) [21]:

\[
\Delta \beta_0 = -\Delta \beta_{NL}.
\] (2)

Commonly, the linear phase mismatch \( \Delta \beta_0 \) is approximated by the formula [22]:

\[
\Delta \beta_0 \approx -\beta_{2,p} \Delta \Omega^2 - \frac{\beta_{4,p}}{12} \Delta \Omega^4 - \ldots .
\] (3)

Here \( \beta_{m,p} = \partial^m \beta_0 / \partial \omega^m |_\omega=\omega_p \) is the \( m \)th order dispersion parameter evaluated at the pump frequency \( \omega_p \), and \( \Delta \Omega = \omega_p - \omega_s \) represents the frequency detuning between the pump and signal waves. Phase-matching can be realized in a waveguide by engineering its geometry [23] so that the dispersion characteristics satisfy (2), (3).

Alternatively, QPM also enables efficient FWM interactions [9]. This term encompasses a set of techniques that counter the detrimental impact of the phase mismatch by (quasi-) periodically varying the medium along the waves’ propagation path. Here we focus on the QPM techniques that rely on adiabatically varying the phase-mismatch value, namely QPM by PMS [9] and QPM by dispersion compensation [11], [24], [25]. Both of these techniques can be realized in any waveguide medium by adiabatically varying the waveguide cross-section along the propagation path. There also exist other QPM techniques, but we do not consider them here as they either require the waveguide medium to have additional properties, such as the possibility to modulate the strength of the FWM interactions [12], [19], [20], or complicate the system considerably by for instance requiring an amplifier chain to obtain periodic power variations [26], [27].

Any QPM technique is best understood in terms of the phase difference \( \Delta \phi = 2\phi_p - \phi_s - \phi_i \) between the interacting waves. The value of \( \Delta \phi \) at any point along the propagation direction \( z \) determines whether the FWM process induces idler gain or loss at that point [9]. The evolution of \( \Delta \phi \) along \( z \) is therefore essential for the FWM process, which is also expressed by the fact that this evolution corresponds to the phase-mismatch \( \Delta \beta \) [9], [21]:

\[
\frac{d\Delta \phi}{dz} = \Delta \beta.
\] (4)

The initial value of \( \Delta \phi \) can be related to the phase \( \Delta \phi_{FWM,i} \) of the complex idler FWM gain [9], [21]:

\[
\Delta \phi (0) = -\Delta \phi_{FWM,i}.
\] (5)
The exact value of the idler complex FWM gain, and thus of \( \Delta \phi_{\text{FWM},i} \), depends on the nature of the nonlinearities involved. However, since this value is not relevant to the current discussion, we do not elaborate on this FWM gain here, but instead refer to Refs. [9], [21] for an explicit definition. The FWM process induces idler gain only for \( \Delta \phi \) values that are located within a range of \( \pi \) around \( \Delta \phi_{\text{FWM},i} + 2n\pi \) [9], i.e., values within \(-\pi/2 < \Delta \phi + \Delta \phi_{\text{FWM},i} + 2n\pi < \pi/2\) (light-grey areas in Fig. 1). For other values, the FWM process results in idler loss (dark-grey areas in Fig. 1). In terms of the phase difference, conventional phase-matching corresponds to maintaining a constant phase-mismatch small enough to ensure that \( \Delta \phi \) remains within the initial gain region (see full line in Fig. 1).

The QPM technique of PMS improves the FWM efficiency by extending the distance over which there is idler gain, while simultaneously reducing the distance over which there is idler loss [9]. This is achieved by alternating the phase-mismatch between two values \( \Delta \beta^+ \) and \( \Delta \beta^- \) that share the same sign \((\Delta \beta^+ \Delta \beta^- > 0)\), and satisfy \(|\Delta \beta^+| < |\Delta \beta^-|\) (see dashed line in Fig. 1). When neglecting the impact of the nonlinear phase mismatch and of optical losses, PMS requires a waveguide with a phase-mismatch profile [9]:

\[
\Delta \beta_0 (z) = \begin{cases} 
\Delta \beta^+_0, & \text{if } -\frac{L^+}{2} \leq z - n(L^+ + L^-) < \frac{L^+}{2}, \\
\Delta \beta^-_0, & \text{if } \frac{L^+}{2} \leq z - n(L^+ + L^-) < -L^- + \frac{L^+}{2}.
\end{cases}
\]

(6)

Here \( \Delta \beta^+_0 \) and \( \Delta \beta^-_0 \) are the phase-mismatch values of the gain and loss sections, respectively, which satisfy the same constraints as the \( \Delta \beta \) values \((\Delta \beta^+_0 \Delta \beta^-_0 > 0 \) and \(|\Delta \beta^+_0| < |\Delta \beta^-_0|\)), and \( L^+ \) and \( L^- \) are the respective sections’ lengths. The latter are related to the former by the necessary condition for PMS:

\[
\Delta \beta^+_0 L^+ = \pm \pi, \quad (7a)
\]

\[
\Delta \beta^-_0 L^- = \pm \pi. \quad (7b)
\]

Alternatively, QPM by dispersion compensation relies on periodically resetting the accumulated phase difference [11], [13], [25]. To do so, every section with a phase-mismatch \( \Delta \beta^+ \) is countered by a consecutive section that has a phase-mismatch \( \Delta \beta^- \) with an opposite sign (see dotted line in Fig. 1). Conventionally, the section lengths are chosen as long as possible [11], [13], [25]. When neglecting in first approximation the impact of the nonlinear phase mismatch and of optical losses, such conventional dispersion compensation can be achieved by a periodic phase-mismatch profile identical to the one of (6), but with linear phase-mismatch values \( \Delta \beta^+_0 \) and \( \Delta \beta^-_0 \) that have opposite signs \((\Delta \beta^+_0 \Delta \beta^-_0 < 0)\) and section lengths \( L^+ \) and \( L^- \) chosen such that they realize a shift in \( \Delta \phi \) of \( \pm \pi \) over each section. However, we point out that QPM by dispersion compensation can also be achieved for shorter section lengths \( L^+ \) and \( L^- \), as long as they satisfy:

\[
\Delta \beta^+_0 L^+ = \rho \pi, \quad (8a)
\]

\[
\Delta \beta^-_0 L^- = -\rho \pi. \quad (8b)
\]

Here \( \rho \) is a factor indicating how large the phase difference \( \Delta \phi \) is allowed to become \((|\rho| = \max (\Delta \phi) / \pi)\), and should satisfy \(|\rho| \leq 1\). Its sign corresponds to the sign of \( \Delta \beta^-_0 \), and indicates whether the phase difference increases or decreases in the initial gain section.

The phase-matching and QPM techniques described allow one to achieve efficient FWM interactions between a single set of pump, signal, and idler waves. We now discuss whether these schemes can also be employed to enable efficient interaction between two arbitrary sets of waves.

### III. Simultaneous QPM of Two Arbitrary Processes

Whether efficient FWM interactions can be achieved between two sets of pump, signal, and idler waves in the same waveguide largely depends on the dispersion characteristics of the waveguide considered. Hence, to continue our analysis, we consider a concrete example of the waveguide media of interest, namely a waveguides characterized by a single zero-dispersion point. Specifically, we consider the TE-mode of an air-clad, rectangular SOI waveguide (see inset Fig. 2(a)). Such SOI nanowaveguides are well-suited for realizing phase-matched [8], [14], [23], [28], [29] and QPM [9], [10] FWM interactions in the near-infrared wavelength domain. As FWM configurations we consider a first set of waves located around a pump at \( \lambda_{p_1} = 1550 \text{ nm} \), and a second set of waves containing a pump at \( \lambda_{p_2} = 1686 \text{ nm} \). This set-up for instance corresponds to the one-to-two Raman wavelength converter considered in Section IV-B.

To phase-match these two FWM configurations simultaneously through dispersion engineering, the SOI waveguide should be engineered in such a way that its dispersion characteristics satisfy (2), (3) for both sets of waves. The dispersion characteristics of a waveguide medium with a single zero-dispersion point are not compatible with such a double-phase-matching scheme. As the height \( h \) is typically set to a fixed value of \( h = 220 \text{ nm} \) when considering silicon photonics foundries [31],
the SOI waveguide has to be dispersion engineered by tuning the width $w$. For example, one can obtain a zero second-order dispersion either near $\lambda_{p_1} = 1550$ nm for $w \approx 755$ nm, or near $\lambda_{p_2} = 1686$ nm for $w \approx 730$ nm (see Fig. 2(a)). Indeed, a zero second-order dispersion simultaneously at both wavelengths as required for a double-phase-matching scheme, is not possible.

An alternative option is a double-QPM scheme. As discussed in Section III, QPM based on either PMS or dispersion compensation can be achieved by alternating adiabatically between two width values $w^+$ and $w^-$ along the propagation length. By choosing the $w^+$ and $w^-$ values such that the two FWM configurations are simultaneously QPM, efficient interactions between both sets of waves could be ensured. Such a scheme would allow for efficient interactions between two arbitrary FWM configurations even in commonly available waveguide geometries with a single zero-dispersion point, such as the rectangular SOI waveguide described above.

To achieve the double-QPM scheme proposed, we have to determine waveguide widths $w$ for which the phase-mismatch values $\Delta \beta_1$ and $\Delta \beta_2$ of the two considered configurations correspond to one of the QPM schemes. To simplify this analysis, it is useful to introduce the phase-mismatch ratio $\rho$ between the two configurations:

$$\rho (w) = \frac{\Delta \beta_1 (w)}{\Delta \beta_2 (w)} \approx \frac{\beta_{2,p_1} (w) \Delta \Omega_1^2}{\beta_{2,p_2} (w) \Delta \Omega_2^2}.$$  

Here the second equality is a first-order approximation obtained by substituting (1) and (3) after neglecting the impact of higher-order dispersion and of the nonlinear phase mismatch.

The conditions for which a double QPM is achieved depend on the exact QPM scheme employed for each set of interacting waves. First, QPM by PMS of both configurations requires (7) to be satisfied for each configuration. Dividing (7a), (7b) expressed for the first configuration by the same equations expressed for the second, leads by means of (9) to the requirement $\rho (w^+) = \rho (w^-) = 1$. Second, to QPM both configurations by dispersion compensation, (8) should be satisfied for each set of waves. Dividing in a similar fashion (8a), (8b) as expressed for the first set of waves by the same equations expressed for the second, results in the requirement $\rho (w^+) = \rho (w^-)$. Third, QPM the first configuration by PMS and the second by dispersion compensation requires the first set of waves to satisfy (7) and the second (8). Dividing (7a), (7b) expressed for the first set of waves by, respectively, (8a), (8b) expressed for the second, leads to the requirement $\rho (w^+) = -\rho (w^-)$ with $|\rho (w^+)| \leq 1$. Note that the reverse scheme, i.e., QPM by dispersion compensation of the first configuration and by PMS of the second, also leads to the same requirement but with $|\rho (w^+)| \leq 1$.

Of the three requirements derived $\rho (w^+) = \rho (w^-) = 1$ (double PMS), $\rho (w^+) = -\rho (w^-)$ (double dispersion compensation), and $\rho (w^+) = -\rho (w^-)$ (combined PMS and dispersion compensation), only the latter is compatible with the dispersion characteristics of the rectangular SOI waveguide considered (see Fig. 2(b)). Indeed, the former two can not be achieved as there are no two $w$ values that correspond to an identical phase-mismatch ratio $\rho$. On the other hand, since $\rho$ varies over a broad range of both positive and negative values, $\rho (w^+) = -\rho (w^-)$ can be met for a large variety of $w^+$ values. The latter implies that the double-QPM scheme by combined PMS and dispersion compensation is not only feasible, but can even be realized with a degree of freedom that allows setting either $w^+$ or $w^-$, or any derived parameter such as the phase-mismatch ratio $\rho$, the shift in width $\Delta w = w^+ - w^-$, or the average of the two widths $w_{0} = (w^+ + w^-)/2$, to a value of choice.

Hence, in waveguide media characterized by a single zero-dispersion point, such as the SOI waveguide considered, efficient FWM interactions between two arbitrary sets of waves can only be achieved by a double-QPM scheme combining both QPM by PMS and QPM by dispersion compensation. The scheme can also be derived from the second-order dispersion characteristics depicted in Fig. 2(a). By choosing a $w^+$ and a $w^-$ value that lie on opposite sides of the width corresponding to a zero second-order dispersion for one of the FWM configurations, QPM by dispersion compensation of that configuration can always be achieved by satisfying (8) via a proper choice of $L^+$ and $L^-$. Moreover, if the $w^+$ and $w^-$ values lead for the other FWM configuration to $\beta_{2,p}$ values with the same sign, then QPM by PMS can be realized for that configuration by instead choosing $L^+$ and $L^-$ so that (7) is satisfied. However, if $\rho (w^+) = -\rho (w^-)$, then (8) for the first configuration already implies (7). In that case, a waveguide designed for achieving QPM by PMS for the first configuration leads automatically also to QPM by dispersion compensation for the second (see dashed and dotted lines in Fig. 1).

Note that the presence of a single zero-dispersion point is a sufficient, but not necessary condition to realize the double-QPM scheme described. Indeed, the variation in phase-mismatch values described above could, under certain conditions, also be realized in the presence of more than one zero-dispersion point, for instance two or three. However, this
be repeated for small shifts in the $w^+$ and $w^-$ parameters, and the waveguide performing best should be selected as the final design. In our example we treat the given $\Delta w$ value as constant, and sweep the mean width $w_0 = (w^+ + w^-)/2$ over a small range of values during this final step. This serves to counter the effects of the higher-order dispersion and the nonlinear phase mismatch that also affect the efficiency of the QPM by dispersion compensation, but were neglected while deriving the requirement $\rho (w^+) = -\rho (w^-)$ for double QPM.

We would like to emphasize that, in the double-QPM scheme proposed here, there is no tradeoff between the two QPM techniques for the different FWM configurations. Instead, the scheme finds a waveguide structure that realizes the full potential of both techniques simultaneously. Hence, the general characteristics, such as the conversion efficiencies and bandwidths, with which these techniques can be realized in the double-QPM scheme are the same as if each of these techniques was realized separately. In silicon waveguides, these characteristics have already been described in literature: QPM by PMS leads to wavelength-selective conversion with an efficiency of $>-23$ dB over a waveguide length of 1 cm and a conversion-efficiency peak near the target signal wavelength of, in angular frequencies, 3–14 THz at near-infrared wavelengths [9]. In contrast, QPM by dispersion compensation results in broadband wavelength conversion with a quasi-symmetrical bandwidth around the pump wavelength that includes the target signal wavelength [25]. For the latter, a bandwidth of more than 300 nm and a conversion efficiency of $>-20$ dB was reported over a length of 1.5 cm [25].

To demonstrate the design method described and illustrate the potential of the double-QPM scheme, we now consider two devices that due to the scheme can be operated at larger pump-signal frequency differences than previously possible, namely the wavelength converter switch and the one-to-two Raman wavelength converter.

### IV. Applications

#### A. Wavelength Conversion Switch

Most conventional wavelength converters based on degenerate FWM are designed to convert a signal wave into a specific idler wave. Only the FWM interactions between the signal wave at $\omega_i$ and one pump wave at $\omega_{p1}$ can be (quasi-)phase-matched, resulting in an idler wave at $\omega_i = 2\omega_{p1} - \omega_s$. If one wants to also convert the signal wave around a second pump at $\omega_{p2}$ to a second idler at $\omega_{i2} = 2\omega_{p2} - \omega_s$, then one needs to design and implement a second such converter.

However, based on the double-QPM scheme, both of these processes can be QPM in the same device. If the resulting device is pumped at $\omega_{p1}$, the signal is converted to the first-idler wave $i_1$, whereas pumping at $\omega_{p2}$ results in conversion to the second-idler wave $i_2$ (see Fig. 4). Hence, switching the pump frequency results in a switching of the output idler frequency. Such a device enables all-optical switching by controlling the pump frequency. As mentioned previously, doing the same with most conventional converters based on degenerate FWM would require two separate converters and thus a much more complicated system. A special type of phase-matched converter, namely one based on an optical fiber that is designed to not...
only display a zero dispersion, but also a zero dispersion slope, can also achieve wavelength conversion for a range of pump wavelengths [16] and thus also realize the wavelength converter switch proposed. However, in such a scheme, the frequency differences for which the wavelength conversion can be achieved are limited, as a conversion bandwidth of 35 nm can typically only be maintained over a pump wavelength range of about 30 nm. As we show below, the double QPM scheme we propose allows much larger frequency differences. Alternatively, controlling the idler frequency by means of a pump frequency can also be achieved by a non-degenerate, double-pump FWM scheme [2]. In this scheme, one pump is placed very closely in frequency to the signal, and the other very closely to the idler desired. Here, the second pump’s frequency directly determines the idler frequency, and switching this pump’s frequency also results in a switching of the idler frequency. However, compared to the wavelength conversion switch we propose, this double-pump scheme requires an additional pump, and also places a stringent requirement on the pump frequencies as they have to be placed very closely to the signal and idler frequencies.

To illustrate the concept, we consider a signal at $\lambda_s = 1750$ nm with an input power of $P_{s,0} = 100 \mu$W that is converted by the nonlinear Kerr-based FWM interactions of a rectangular SOI waveguide. The signal is either converted around a pump at $\lambda_{p_1} = 1600$ nm and with input power $P_{p_1,0} = 300$ mW to an idler at $\lambda_{i_1} = 1474$ nm, or around a pump at $\lambda_{p_1} = 1550$ nm with input power $P_{p_1,0} = 300$ mW to an idler at $\lambda_{i_2} = 1391$ nm. To simulate the nonlinear propagation of these waves, we solve the set of coupled, first-order differential equations for the waves’ complex amplitudes described in Ref. [9]. The propagation equations model the impact of the various nonlinear effects present in silicon, including self- and cross-phase modulation, two-photon-absorption (TPA), the free-carrier index change, free-carrier absorption (FCA), and Kerr-based FWM. Following the same reference, we first compute the dispersion parameters and the relevant nonlinear mode overlap factors over a range of $w$ values with the commercial eigensolver MODE Solutions [30], and fit a polynomial in $w$ to the data calculated for each width-dependent parameters. These polynomials allow us to simply evaluate the parameters for any $w$ value and to solve the propagation equations for any width profile $w(z)$. The linear loss is assumed independent of $w$ and constant over the considered wavelength domain with a value $\alpha = 1$ dB/cm in agreement with measurements in SOI nanowaveguides [23]. For the nonlinear material parameters of silicon we employ the values given in Ref. [9], and which are also discussed in the appendix.

In line with the discussion in Section III, the two considered FWM processes can not be simultaneously phase-matched by tuning the width $w$ of the rectangular SOI waveguide described in Section III (see Fig. 5(a) in which we approximated the phase-mismatch $-\Delta \beta \approx \beta_{2,p} \Delta \Omega^2$ in first order according to (3)). On the other hand, calculating the phase-mismatch ratio $\rho(w)$ with (9) indicates that the requirement for double QPM $\rho(w^+) = -\rho(w^-)$ can be satisfied for a variety of $w^+$ values (see Fig 5(b)). A shift in width of for instance $\Delta w = 80$ nm allows satisfying the double-QPM requirement for a mean width $w_0 \approx 813$ nm (corresponding to section widths $w^+ = 773$ nm and $w^- = 853$ nm). Based on these parameters, we can generate the width profile of a double-QPM waveguide dynamically based on (10) while simulating the nonlinear propagation of the set of waves QPM by PMS, i.e., of $s$, $p_1$, and $i_2$. Here the different constant-width sections are connected with linear tapers of 25 $\mu$m to guarantee an adiabatic variation of the waveguide geometry [9], and these linear tapers are included in the simulations. Subsequently simulating the propagation of $s$, $p_1$, and $i_1$ allows one to determine the design’s performance.

Fig. 4. Two operating modes of the wavelength converter switch: (a) when pumping at a frequency $\omega_{p_1}$, the signal at $\omega_s$ is converted to an idler at $\omega_{i_1}$, and (b) and when pumping at $\omega_{p_2}$ the same signal is converted to an idler at $\omega_{i_2}$.

Fig. 5. Tuning the width $w$ of a rectangular SOI waveguide with height $h = 220$ nm does (a) not allow to obtain a zero phase-mismatch $\Delta \beta_0$ simultaneously for the two FWM processes of the proposed wavelength conversion switch, but (b) does allow to satisfy the condition on phase-mismatch ratio $\rho$ for double-QPM ($\rho(w^+) = -\rho(w^-)$). The markers in (b) indicate the width values for which $\rho(w^+) = -\rho(w^-)$ is satisfied with $\Delta w = w^+ - w^- = 80$ nm.
to account for the nonlinear phase mismatch and higher-order dispersion, we also sweep $w_0$ to optimize the design’s performance.

The design thus obtained has a mean width of $w_0 = 811$ nm corresponding to section widths $w^+ = 771$ nm and $w^- = 851$ nm (see Fig. 6(a)). After 2 cm, conversion efficiencies $P_{1_s}/P_{s,0}$ and $P_{2_s}/P_{s,0}$ of more than $-15$ dB are achieved for both modes of operation (see Fig. 6(b)). These efficiencies are significantly better than those reported for both numerical simulations and experiments of the non-degenerate double-pump scheme [2] we discussed previously and which can possible realize the same functionality as the converter switch proposed. The power evolution $P_{s,0}$ displays a continuous increase with propagation length which indicates a FWM process that is QPM by dispersion compensation. The second-idler power $P_{2_s}$ on the other hand increases in the gain sections ($w = w^+ = 771$ nm), but decreases in the loss sections ($w = w^- = 851$ nm), as is characteristic for a FWM process QPM by PMS. This confirms that the design indeed realizes a double-QPM scheme which leads to efficient interactions for both FWM configurations.

Note that the pump-signal frequency differences considered here are not the maximum values for which a wavelength converter switch could be designed. Indeed, since QPM by PMS can be used to realize efficient conversion throughout the entire near-infrared wavelength domain of 1300–1900 nm [9] with similar SOI waveguides as the ones considered here, we expect that the wavelength converter switch could similarly be designed for wavelengths throughout this near-infrared domain. As long as efficient light propagation is supported, even larger frequency differences, which correspond to operation in the mid-infrared wavelength domain, could be feasible, but such conditions fall outside the scope of this paper.

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**B. One-to-Two Raman Wavelength Converter**

Up to now, we have focused on the generation of an idler at a frequency $\omega_i = 2\omega_p - \omega_s$ from a weak signal wave interacting with a strong pump wave. This generation is based on the degenerate FWM process that involves two pump photons, one signal and one idler $i$ photon. However, FWM interactions between the pump and signal also give rise to a second idler or satellite wave at a frequency $\omega_j = 2\omega_s - \omega_p$ (see Fig. 7). This second idler results from another degenerate FWM process that involves two signal photons, one pump and one second-idler $j$ photon. Generation of such a second-idler wave has been observed as a side effect in experiments aimed at generating a conventional idler wave $i$ by degenerate FWM in SOI nanowaveguides [32], ring resonators [1], and photonic crystal waveguides [33]. In all these experiments, no specific measures were taken to counter the phase-mismatch $2\beta_{1,s} - \beta_{0,p} - \beta_{0,j} \approx -\Delta \Omega^2$ of the secondary FWM process. However, the observed second-idler build-up can be explained by the very small pump-signal frequency detunings (1–6 nm) common to all these experiments. For a small frequency detuning $\Delta \Omega$, the pump and signal second-order dispersion parameters differ little since $\beta_{2,s} \approx \beta_{2,p} - \beta_{2,j} \Delta \Omega$. Hence, the phase-matching of the primary FWM process generating the idler $i$ implied in these experiments also phase-matching of the secondary FWM process generating the idler $j$.

However, for larger frequency detunings, the discussion of Section III becomes relevant. In terms of that section, the waves $p$, $s$, and $i$ can be seen as first FWM configuration $p_1$, $s_1$, and $i_1$, and the waves $s$, $p$, and $j$ as a second FWM configuration $p_2$, $s_2$, and $i_2$. Efficient generation of both idlers $i$ and $j$, and thus one-to-two wavelength conversion, can then only be generated by employing a double-QPM scheme that QPM both configurations.

In literature, several other schemes have been previously proposed to achieve one-to-two [2] and even one-to-four wavelength conversion [4]. All these schemes are based on non-degenerate, double-pump FWM for very small frequency detunings between the signal and (one of) the pumps. In contrast, the one-to-two wavelength converter we propose does not only require just a single pump to operate, which simplifies the device’s complexity considerably, but also provides more design freedom as it can be implemented for any pump-signal frequency detuning. For instance, it allows us to realize a silicon-based one-to-two wavelength converter operating at Raman resonance, and potentially benefit from the additional nonlinear interactions offered by the strong, but resonant Raman scattering effect in silicon [28], [34]. To illustrate this, we consider
a rectangular SOI waveguide with as input a signal wave at \( \lambda_s = 1686 \text{ nm} \) with power \( P_{r,0} = 100 \mu \text{W} \), and a pump wave at \( \lambda_p = 1550 \text{ nm} \) with power \( P_{p,0} = 300 \mu \text{W} \). The corresponding idler wavelengths are \( \lambda_i = 1434 \text{ nm} \) and \( \lambda_j = 1848 \text{ nm} \). Such a configuration corresponds to on-Raman-resonance operation in silicon [28]. We simulate the nonlinear propagation of these waves by the same method described in the previous section, but with the nonlinear propagation equations extended to four coupled, first-order differential equations that take into account the additional nonlinear interactions, including Raman scattering and non-degenerate FWM. The exact form of these extended equations is not relevant for the current discussion, and is thus discussed in the appendix.

As discussed in Section III and similar to Section IV-A, the two degenerate-FWM processes generating both idlers can once again not be simultaneously phase-matched by tuning the waveguide width \( w \) (see Fig. 8(a)). However, the corresponding phase-mismatch ratio \( \rho(w) \) indicates that the requirement for double QPM \( \rho(w^+)=\rho(w^-) \) can be satisfied for a wide range of \( w^+ \) values (see Fig 8(b)). Here we depicted the phase-mismatch ratios \( \beta_{2,s}/\beta_{2,p} \) and \( \beta_{2,p}/\beta_{2,s} \), separately, as they are related with different double QPM schemes. Values smaller than one for the former lead to double QPM with QPM by PMS realized for the FWM process generating idler \( i \), whereas values smaller than one for the latter lead to double QPM with QPM by PMS for the process generating idler \( j \). For a shift in width of \( \Delta w = -80 \text{ nm} \), the double-QPM requirement can be realized for a mean width \( w_0 \approx 846 \text{ nm} \) (corresponding to section widths \( w^+ = 806 \text{ nm}, w^- = 886 \text{ nm} \)). For these parameters, the FWM process generating the first idler \( i \) can be QPM by PMS, and the one generating the second idler \( j \) by dispersion compensation. To generate the corresponding double-QPM waveguide, we simulate the nonlinear propagation of all four waves simultaneously (see the appendix). During this simulation, we obtain the width profile dynamically by evaluating (10) based on the sign of \( dP_i/dz \), as it is the first idler that is generated by PMS. As in the previous section, the different constant-width sections are connected by 25 \( \mu \text{m} \)-long linear tapers. As the propagation of the four waves is simulated simultaneously, the same simulation also provides the performance of the second-idler generation. Finally, we optimize this performance by repeating the same dynamic width-profile generation for different \( w_0 \) values and then selecting the best design.

The optimized design has a mean width of \( w_0 = 843 \text{ nm} \) corresponding to section widths \( w^+ = 883 \text{ nm} \) and \( w^- = 803 \text{ nm} \) (see Fig. 9(a)). The second-idler conversion efficiency \( P_j/P_{r,0} \) surpasses -20 dB after less than 2 cm, and -10 dB after less than 4 cm (see Fig. 9(b)). After 5 cm, a second-idler conversion efficiency of \( P_j/P_{r,0} \approx -7.3 \text{ dB} \) and a first-idler conversion efficiency of \( P_i/P_{r,0} \approx -1.9 \text{ dB} \) is achieved. These efficiencies outperform those simulated and observed for the one-to-two wavelength converter based on non-degenerate double-pump FWM with small pump-signal frequency detunings [2]. Note that the first-idler generation is more efficient because the corresponding FWM process scales more strongly with the pump power. Indeed, the first-idler generation relies on a FWM process that involves two pump photons, whereas the second-idler generation originates from a FWM process that involves only one pump photon. However, it should be noted that the conversion efficiencies simulated for both idlers lie much closer together than the efficiencies observed for second-idler generation with small pump-signal frequency detunings [1], [32], [33]. The first-idler power evolution \( P_i \) also displays the distinct gain and loss regions that are characteristic for QPM by PMS. On the other hand, that of the second idler \( P_j \) shows the monotonic
increase corresponding to QPM by dispersion compensation. Hence, the simultaneous efficient generation of both idlers is indeed realized by double QPM.

V. CONCLUSION

We developed a double-QPM scheme that enables simultaneous QPM of two arbitrary FWM processes. The scheme combines two QPM techniques, namely QPM by PMS for one process with QPM by dispersion compensation for the other. We showed that this is the only scheme that can enable efficient interactions for two FWM processes in essentially any type of waveguide medium that is characterized by a single zero-dispersion point in the wavelength domain of interest, but not by any other necessary properties. The scheme does not require extensive dispersion engineering of the waveguide geometry, but instead enables efficient interactions by adiabatically alternating between two waveguide geometries along the waveguide length.

To illustrate the potential of the scheme, we introduced two applications that due to the double-QPM scheme can be operated at larger pump-signal frequency differences than previously possible, namely a wavelength conversion switch and a one-to-two Raman wavelength converter. The former device allows one to switch between two output idler frequencies by switching the pump frequency, while the latter provides the generation of two idlers, rather than one, out of the Raman-resonant interactions between a weak signal and strong pump. By means of numerical simulations, we showed that both of these devices can be realized by implementing the double-QPM scheme with a SOI nanowaveguide structure for which the waveguide width was adiabatically varied, resulting in conversion efficiencies of $\approx -15$ dB for both idlers in the wavelength conversion switch and a first- and second-idler conversion efficiency of $-1.9$ and $-7.3$ dB respectively in the one-to-two Raman converter. This shows that the scheme developed indeed enables efficient interactions between two arbitrary FWM processes in any type of waveguide medium that is characterized by a single zero-dispersion point.

Hence, double QPM allows optical waves to interact through a larger number of nonlinear interactions. By enriching the variety of possible optical nonlinearities, it leads to novel FWM-based nonlinear devices.

APPENDIX

PROPAGATION EQUATIONS FOR ONE-TO-TWO RAMAN WAVELENGTH CONVERTERS

The continuous-wave light propagation in the one-to-two Raman wavelength converter described in Section IV-B can be

$$\frac{dA_p}{dz} = \left( i\dot{A}_{0,p} - \frac{\alpha_p}{2} \right) A_p + \sum_{l=p,s,i,j} \left( i\gamma_{K,p} (2 - \delta_p) \Gamma_{Ipp}^K + i\gamma_{R,p} \Gamma_{Ipp}^R \right) S_l S_p |A_i|^2 A_p + \left( \frac{\omega_p}{c} n_{f,p} - \frac{\alpha_{f,p}}{2} \right) \Gamma_{Ipp}^N S_p A_p$$

$$+ i\gamma_{R,s} \sum_{l=p,s,i,j} \Gamma_{Ipp}^R H_R (\omega_p - \omega_i) S_l S_p |A_i|^2 A_p + i \left( \gamma_{K,s} \Gamma_{Ips}^K + \gamma_{R,s} \Gamma_{Ips}^R \right) S_p S_s |A_j|^2 A_p$$

$$+ i \left( \sqrt{S_p S_s A_i A_j} \Gamma_{Ips}^N \right)$$

$$\frac{dA_s}{dz} = \left( i\dot{A}_{0,s} - \frac{\alpha_s}{2} \right) A_s + \sum_{l=p,s,i,j} \left( i\gamma_{K,s} (2 - \delta_s) \Gamma_{Isp}^K + i\gamma_{R,s} \Gamma_{Isp}^R \right) S_l S_s |A_i|^2 A_s + i \left( \gamma_{K,s} \Gamma_{Ips}^K + \gamma_{R,s} \Gamma_{Ips}^R \right) S_p S_s |A_j|^2 A_s$$

$$+ i \left( \sqrt{S_p S_s A_i A_j} \Gamma_{Ips}^N \right)$$

$$\frac{dA_j}{dz} = \left( i\dot{A}_{0,j} - \frac{\alpha_j}{2} \right) A_j + \sum_{l=p,s,i,j} \left( i\gamma_{K,j} (2 - \delta_j) \Gamma_{Ijp}^K + i\gamma_{R,j} \Gamma_{Ijp}^R \right) S_l S_j |A_i|^2 A_j + \left( \frac{\omega_j}{c} n_{f,j} - \frac{\alpha_{f,j}}{2} \right) \Gamma_{Ijp}^N S_j A_j$$

$$+ i \left( \sqrt{S_p S_s A_i A_j} \Gamma_{Ijp}^N \right)$$

$$\frac{dA_i}{dz} = \left( i\dot{A}_{0,i} - \frac{\alpha_i}{2} \right) A_i + \sum_{l=p,s,i,j} \left( i\gamma_{K,i} (2 - \delta_i) \Gamma_{Iii}^K + i\gamma_{R,i} \Gamma_{Iii}^R \right) S_l S_i |A_j|^2 A_i + \left( \frac{\omega_i}{c} n_{f,i} - \frac{\alpha_{f,i}}{2} \right) \Gamma_{Iii}^N S_i A_i$$

$$+ i \left( \sqrt{S_p S_s A_i A_j} \Gamma_{Iii}^N \right)$$
simulated by solving a set of four coupled, first-order differential equations. These equations describe the evolution of the complex amplitudes of the involved waves along the waveguide and can be derived by general methods described in literature [6], [35]. Since these equations are less common in literature than those employed in Section IV-A describing nonlinear interactions between three coupled waves, we briefly discuss them here.

The nonlinear continuous-wave light propagation of the pump \( p \), signal \( s \), first idler \( i \), and second idler \( j \) waves in a SOI waveguide can be described by the four coupled equations of (11)–(14) as shown on the bottom of the previous page. In these equations we employ a similar notation as in Ref. [9] in which \( A_l \) with \( l = p, s, i, j \) represent the complex amplitudes of, respectively, the pump, signal, first-idler, and second-idler waves. \( \beta_l \) is the dispersion parameter (see Section II) and \( \alpha_l \) is the linear loss coefficient, here assumed to be \( \alpha_l = 1 \) dB/cm [23]. The coefficients \( \gamma_{K,l} \) and \( \gamma_{R,l} \) describe the nonlinear Kerr and Raman effects, respectively. \( \gamma_{K,l} \) is related to the Kerr coefficient \( n_2 = 6 \times 10^{-5} \) cm\(^2\)/GW and the coefficient for TPA \( \beta_T = 0.45 \) cm/GW in the near-infrared wavelength domain [22], whereas \( \gamma_{R,l} \) depends on the Raman shift \( \Omega_R = 2 \pi \times 15.6 \) THz, the Raman linewidth \( \Gamma_R = 2 \pi \times 52.5 \) GHz [22], [34], and the Raman gain \( g_{R,\text{ref}} = 20 \) cm/GW at the reference frequency \( \omega_{\text{ref}} = 2\pi c/1542.3 \) nm [36]:

\[
\gamma_{K,l} = \frac{\omega_l}{c} n_2 + i \frac{\beta_T}{2}, \quad \gamma_{R,l} = \frac{\omega_l}{\omega_{\text{ref}}} g_{R,\text{ref}} \frac{\Gamma_R}{\Omega_R}. \tag{15}
\]

\( H_R \) is the Raman spectral response which is often approximated by a Lorentz shape of the form [35]:

\[
H_R (\Delta \Omega) = \frac{\Omega_R^2}{\Omega_R^2 - \Delta \Omega^2 - 2i \Gamma_R \Delta \Omega}. \tag{16}
\]

The coefficients \( \alpha_{f,l} \) and \( n_{f,l} \), describe, respectively, the FCA and the free-carrier index change. Around a reference wavelength \( \lambda_r = 1550 \) nm, they are commonly related to the free-carrier density \( N \) by means of two empirical formulas [35], [37]:

\[
\alpha_{f,l} = \left( \frac{\omega_l}{\omega_i} \right)^2 14.5 \times 10^{-18} N, \tag{17}
\]

\[
n_{f,l} = \left( \frac{\omega_l}{\omega_i} \right)^2 (-8.8 \times 10^{-4} N - 8.5 N^{0.8}) \times 10^{-18}. \tag{18}
\]

In the continuous-wave regime, the density \( N \) generated by TPA can in turn be related to the carrier lifetime \( \tau_0 \), which typically has a value of 1 ns in SOI nanowaveguides [22], [38], and to the area of the waveguide \( A_{\text{wg}} = h \omega \) [9] by the formula:

\[
N = \sum_{l,k = p,s,i,j} \tau_0 (2 - \delta_{lk}) \beta_T S_l S_i \Gamma_{ikk} |A_l|^2 |A_k|^2 \frac{h (\omega_l + \omega_k)}{\omega_i} A_{\text{wg}}. \tag{19}
\]

The coefficients \( \Gamma_{K,l,m,n} \), \( \Gamma_{R,l,m,n} \), and \( \Gamma_{p}^R \) are, respectively, the Kerr, Raman, and free-carrier-induced overlap factors [9]. These factors describe the impact of the various waves’ mode profiles on each nonlinear effect. Finally, the factors \( S_l = c/v_{g,l} m_S l \) represent the ratio of the modal phase velocities to the modal group velocities [9], which are also related to the ratios of the modal energy densities to the modal optical power flows [9].

These equations include the FWM interactions induced by both the Kerr and Raman nonlinear effects. More specifically, they include degenerate Kerr FWM between \( p, s, i \) and \( j \) (terms in \( \Gamma_{pji}^R \)) and between \( s, j \) and \( p \) (terms in \( \Gamma_{jsp}^R \)), non-degenerate Kerr FWM between all four waves (terms in \( \Gamma_{pji}^R \)), coherent anti-Stokes Raman scattering between \( p, s, i \), and \( j \) (terms in \( \Gamma_{pjs}^R \)) and between \( s, j \), and \( p \) (terms in \( \Gamma_{jsp}^R \)), and the Raman-resonant interaction between all four waves (terms in \( \Gamma_{pji}^R \)). It should be noted that (11)–(14) only consider Raman scattering involving a single phonon. Multi-phonon Raman scattering, through which for instance the pump and second idler might interact, is neglected. This is justified by the fact that second-order Raman scattering peak associated with two transverse optical phonons is not only more than 50 times weaker than the first-order peak Raman peak, but also has its maximum at \( \Delta \Omega = 2 \Omega_R - 8 \) THz [35], [39], [40] which is well below the frequency detuning \( \omega_p - \omega_j = 2 \Delta \Omega_R \) between the pump and second idler.

\textbf{REFERENCES}


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