Identification of modal strains using sub-microstrain FBG data and a novel wavelength-shift detection algorithm

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ABSTRACT

The presence of damage in a civil structure alters its stiffness and consequently its modal characteristics. The identification of these changes can provide engineers with useful information about the condition of a structure and constitutes the basic principle of the vibration-based structural health monitoring. While eigenfrequencies and mode shapes are the most commonly monitored modal characteristics, their sensitivity to structural damage may be low relative to their sensitivity to environmental influences. Modal strains or curvatures could offer an attractive alternative but current measurement techniques encounter difficulties in capturing the very small strain (sub-microstrain) levels occurring during ambient, or operational excitation, with sufficient accuracy. This paper investigates the ability to obtain sub-microstrain accuracy with standard fiber-optic Bragg gratings using a novel optical signal processing algorithm that identifies the wavelength shift with high accuracy and precision. The novel technique is validated in an extensive experimental modal analysis test on a steel I-beam which is instrumented with FBG sensors at its top and bottom flange. The raw wavelength FBG data are processed into strain values using both a novel correlation-based processing technique and a conventional peak tracking technique. Subsequently, the strain time series are used for identifying the beam’s modal characteristics. Finally, the accuracy of both algorithms in identification of modal characteristics is extensively investigated.

1. Introduction

Vibration-Based Structural Health Monitoring, usually abbreviated as VBSHM or VBM, can be a successful approach for damage identification and structural condition assessment of civil structures, e.g. bridges, dams and tunnels [1,2]. A drawback of the method is that currently it suffers from low sensitivity of the eigenfrequencies to certain types of damage, especially to moderately severe local damage [3]. Moreover, the influence of the environmental factors (e.g. temperature) on eigenfrequencies and mode shapes can be high enough to completely mask the presence of damage [3–5]. In contrast, modal characteristics obtained from dynamic strain measurements, such as modal strains and modal curvatures, are much more sensitive to local damage [6,7]. The introduction of fiber-optic sensing systems, that can accurately measure dynamic strains while also offer ease of installation, resistance in harsh environment and long-term stability, contributed to an increased interest in adopting these systems for VBSHM applications [8]. In the past years, fiber-optic sensors like SOFO [8] have been successfully used in operational modal analysis for SHM [9,10]. In this

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context, the present work explores a novel optical signal processing technique that enables to obtain sub-microstrain accuracy with standard Fiber-optic Bragg Grating (FBG) [11] strain sensors in a dense grid, to accurately capture the modal strain shapes.

FBG sensors recently gained interest for the advantages they can offer for vibration measurements and structural health monitoring [12–14]. An important advantage is the ability to multiplex several gratings in a single fiber, offering simultaneous measurements of a large number of sensors, rather than a single one as the SOFO optic sensors [8], which makes them suitable for monitoring of large civil structures. FBGs have been used for real-time monitoring of railway infrastructures [15], for novel structures like the Kaeng Krachan Elephant Shelter of the Zurich Zoo [16], for monitoring of bridge structures, e.g. the Tsing Ma Bridge in Hong Kong, where dynamic strains imposed by traffic were measured but only a few sensors were used while the order of magnitude was tens of microstrains [17]. Furthermore, various studies have been conducted with the implementation of FBGs for experimental modal analysis and system identification but artificial excitation of high amplitude is required for these methods to be applied [18].

The current challenge for the VBSHM of civil structures is to find monitoring systems that are easily implemented over large areas, are sensitive to local damage, while are also cost-effective [19]. In this context, this paper investigates the implementation of FBG sensors for dynamic strain measurements in a dense grid, for system identification and damage localization, as a good trade-off solution to challenge the state-of-the-art. The aim of the study is to directly measure the very small dynamic strains that occur in civil structures, such as bridges, during operational or ambient excitation and to identify the system characteristics from this data. The identification of torsional modes from longitudinal strain measurements is also attempted. A novel correlation algorithm, the Fast Phase Correlation algorithm or FPC [20], is applied to obtain sub-microstrain accuracy and precision in wavelength-shift detection while its performance is compared with a conventional peak-shift detection algorithm, the Maximum Detection Algorithm or MDA [21].

As test structure, a steel I-beam of type IPE 100 is considered which is monitored by two chains of multiplexed FBG sensors and conventional uniaxial accelerometers. The beam was excited with a shaker at low force amplitudes, with systematically decreasing level, to explore the ability to identify modal characteristics from very low Signal to Noise Ratio (SNR) data. Three modal analyses were conducted, using different response data. Accelerometer data were used for the first analysis. The second analysis was conducted with strain data measured by FBG sensors. The third analysis was a combination of the first two; therefore data from both accelerometers and FBG sensors were used for the combined data modal analysis.

The outline of the paper is as follows. Section 2 introduces to the experimental setup that was used for the conducted experiments. The sensors that were used as well their setups are briefly presented. In Section 3, the identification algorithms are briefly discussed while in Section 4 the Finite Element model of the beam that was constructed in ANSYS is presented. Sections 5, 7 and 8 describe the obtained results of the conducted modal analyses and the comparisons between the various applied methods. In Section 6, the basic principles of the peak-shift algorithms, FPC and MDA, are discussed. Finally, in Section 9, the work is concluded.

2. Experimental setup

As test structure, a steel I-beam of type IPE 100 from the European Standard Universal I-Beams is considered (Fig. 1a). The beam has a length of 3.0 m and steel rectangular end plates are welded at its ends with dimensions of 200×200×10 mm. The beam is
suspended at its ends with flexible springs to approximate free–free boundary conditions (Figs. 1–5).

Two kinds of sensors were used to record the response of the beam to the input force signal: Fiber-optic Bragg Grating (FBG) strain sensors and accelerometers. An electrodynamic shaker was used for the excitation of the beam (Fig. 1a). The excitation point coincided with the accelerometer at location 17 (Fig. 5) and a force sensor was also used at this location to measure the input force. The location of the excitation point was selected in a way that would ensure the excitation of both the torsional and the bending about the major axis modes. The beam was excited with periodic random and swept sine signals of various force amplitudes (Root Mean Square (RMS) force amplitude varying from 13.5 to 0.4 N) and with an excitation bandwidth of [0:200] Hz.

2.1. Strain sensors

Two chains of multiplexed FBG strain sensors were attached at one side of the upper and lower flanges of the beam along its longitudinal direction, to measure the axial dynamic strains in the x-direction, as shown in Figs. 1b, 2 and 3. The fibers were glued on the beam at the locations of the FBG sensors (Fig. 1b). The top fiber contained 9 FBGs while the bottom one 8, since at the moment of the experiment the available fibers had different amount of sensors. The distance between two consecutive sensors is about 40 cm. A FBGS FBG-SCAN 700 acquisition system was used for measuring the dynamic strains. The sampling frequency was selected to be $f_s = 950$ Hz, which meets Shannon’s theorem of $f_s \geq 2f_{\text{max}} = 400$ Hz [22], where $f_{\text{max}}$ is the maximum excitation frequency in the bandwidth of interest.

2.2. Accelerometers

In order to accurately capture the mode shapes of the beam, 34 locations were selected to be instrumented on its top flange. Uniaxial accelerometers were fixed in two different measurement setups of 26 sensors per setup on the edges of the top flange of the beam, to measure the vertical accelerations (z-direction) as shown in Figs. 4 and 5. A National Instruments (NI) PXI-1050 chassis acquisition system was used. The original sampling frequency was $f_s = 20,000$ Hz. However, the data were re-sampled afterwards in a lower frequency of $f_s = 1000$ Hz as it was adequate for meeting Shannon’s theorem of $f_s \geq 2f_{\text{max}} = 400$ Hz.

2.3. Systems’ synchronization

Since a combined modal analysis using both strain and acceleration data is one of the aims of this research, the synchronization of the two acquisition systems was obligatory to avoid a phase shift. Therefore, the sampling frequency for the combined modal analysis was $f_s = 950$ Hz, the same as for the FBG sensors. To obtain the same sampling frequency, the accelerometer data were down-sampled to 950 Hz. A trigger signal was commonly sampled by both acquisition systems.

3. System identification

The system identification was conducted with MACEC, a MatLab toolbox for experimental and operational modal analysis [23]. Two identification techniques were applied, the covariance-driven Stochastic Subspace Identification (SSI-cov), which is an output only identification technique [24,25], and the Combined deterministic-stochastic Subspace Identification (CSI) [26], where both the input force and the output measured data (accelerations, strains, etc.) are used for the identification. The SSI-cov algorithm provides also with the 95% confidence interval ($\sigma$) for quantifying the uncertainty of the identified modal characteristics. In both applied techniques, the system identification is performed in the time domain.

4. Finite element model (FEM)

A finite element model (FEM) of the beam was built in ANSYS for comparison with the modal characteristics obtained from the processing of the experimental data. Both the IPE beam and the end plates were simulated with solid elements. Specifically, the element type SOLID 45 was selected from the ANSYS element library which allows to model straightforwardly the whole volume of the beam. SOLID 45 is a linear eight-node element with three degrees of freedom at each node (translations in the nodal $x$-, $y$-, and $z$-direction). The finite element mesh that was employed in the modal analysis was chosen based on the following criteria: (a) the geometry of the structure would be captured as accurately as possible, (b) the finite element shapes would not be over-distorted and (c) the computed eigenfrequencies of interest would not change considerably with additional mesh refinement. Based on these criteria, the chosen finite element mesh size was about 3 mm, which provides the necessary accuracy for the research objectives. Free–free boundary conditions were applied to the model as in the original structure. The material properties that were used, after the model updating, are summarized in Table 1.

The eigenfrequencies that were calculated in the bandwidth of interest, [0:200] Hz, from the FEM are presented in Table 2. The
bending about the major axis modes are noted with ‘B’, the lateral with ‘B-L’ while the torsional modes with ‘T’. It should be mentioned here that the lateral bending modes 1, 3, 6 and 9 are not expected to be identified by the modal test since the beam is excited in bending mainly along its strong axis (z-axis in Fig. 2).

5. Accelerometer data modal analysis

For the first modal analysis only the data obtained from the uniaxial accelerometers were used. The system identification was performed with both the SSI-cov and the CSI techniques. The maximum model order and the half number of Hankel’s block rows \(i\) that have to be defined in both algorithms were selected in a way that would satisfy the rules of thumb that are proposed in the literature [25,26].

In Table 3, the selected values for the aforementioned quantities are summarized. Since the CSI algorithm is a deterministic-stochastic algorithm, high accuracy can be obtained with lower system order. Furthermore, the computational time for the CSI algorithm is quite higher than the SSI-cov algorithm, forcing the user to choose a lower system order with respect to the SSI-cov algorithm.

The modal analyses that were conducted for all the available test data (various force excitation amplitudes and excitation signals) indicate a very good repeatability and further, no significant differences were observed in the results when tests with different force amplitudes and excitation signals are compared. This is valid also for the two setups that were used. Therefore, the obtained dynamic characteristics (eigenfrequencies and mode shapes) from one test which was excited with a periodic random signal in the bandwidth of [0:200] Hz is presented. The RMS amplitude of the input force of the test was 2.37 N.

5.1. Eigenfrequencies

In the bandwidth of interest, 6 modes were identified. The lateral bending modes have not been identified since the accelerometers were measuring only in the vertical direction of the beam (z-axis in Fig. 4). In Table 4, the eigenfrequency values and damping ratios of the identified modes with SSI-cov and CSI are presented. All modes were selected from stabilization diagrams and have MPC values [22] higher than 0.95, meaning that the identified modes are almost purely real. The calculated standard deviation of the eigenfrequency values and the damping ratios, which represents the variance error of the output-only identification with SSI-cov [27], is also presented.

The differences in the eigenfrequency values between SSI-cov and CSI are minor and less than 1.0%. Moreover, the standard deviation is in all cases smaller than 0.11 Hz, indicating a high identification accuracy. When the values in Table 4 are compared with the ones computed with the FEM in Table 2, a good match is observed, with the relative difference for the various modes lower than ±1.5%. Overall, it can be concluded that the eigenfrequency values are well identified from the experimental accelerometer data.

<table>
<thead>
<tr>
<th>Table 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel properties applied in the FE model.</td>
</tr>
<tr>
<td>Young's modulus (E)</td>
</tr>
<tr>
<td>200 GPa</td>
</tr>
</tbody>
</table>
5.2. Mode shapes

The mode shapes of the first six identified modes, as obtained from the SSI-cov identification, are presented in Fig. 6. The mode shapes are smooth and do not pose any significant amplitude inconsistencies. Furthermore, the experimentally determined mass normalized mode shapes with the CSI algorithm are compared with the ones obtained from the FEM in Fig. 7.

In Fig. 7 a good match can be observed between the CSI and the FEM identified mass normalized mode shapes. The CSI identified mode shapes are quite smooth and the few inconsistencies that are observed in the middle of the bottom subplot in Figs. 7b, c and f are probably related to inaccurate sensitivities of the sensors at these locations while they do not seem to affect the results. The implementation of the Modal Assurance Criterion (MAC) between the SSI-cov and the FEM mode shapes, as a measure of consistency (degree of linearity) [28], contributes to the conclusion of the high consistency of the mode shapes. The MAC values, when the same mode shapes are compared, are approximating unity (>0.98) for all modes, proving the high consistency of the shapes.

6. Peak-shift algorithms

When dynamic strains are measured, the ability to determine the Bragg wavelength shift with adequate accuracy and precision is essential. The accuracy is defined as the proximity of the measurements to the actual values while the precision is defined as the degree to which repeated measurements under unchanged conditions show the same results [29]. In Fig. 8, the evolution of the wavelength for four FBG sensors is simulated. Fig. 8a shows the actual wavelengths while Fig. 8b the wavelengths as they would resolve from an acquisition system with low resolution. In this case, the wavelength peaks are resolved by only a few points (Fig. 9) and consequently they are not well identified. Therefore, the wavelength shift detection becomes hard, leading to low accuracy and precision of the obtained results.

When the measured wavelength shifts are lower than the resolution of the acquisition system, the implementation of a peak-shift algorithm that will interpolate the wavelength spectrum down to the desired resolution level is mandatory. In the current section, the principles of two peak-shift algorithms, which were used in the described experiment to increase the spectral resolution of the FBGS interrogator from 90 pm (hardware resolution) to 1 pm or 0.8 μ, are discussed. The first is a conventional peak-shift detection algorithm, the Maximum Detection Algorithm (MDA) and the second a phase correlation algorithm, the Fast Phase Correlation algorithm (FPC).

Conventional peak-shift detection techniques, such as the MDA, can yield inaccurate and imprecise results, especially when the Signal to Noise Ratio (SNR), which is defined as signal power divided by noise power [30], is low and the wavelength resolution of the hardware is limited [21]. To overcome these problems, the FPC detection algorithm, which computes the wavelength shift in the reflected spectrum of a FBG sensor was developed. The main advantages of the new algorithm over the MDA are its low sensitivity to the noise existing in the data and also that is less influenced by the wavelength resolution and the peak locking effect [20]. The peak locking effect consists of a modulation of both precision and accuracy errors, with minimum errors occurring at integer resolution positions, and maximum errors biased toward mid-resolution positions.

The performance of the two algorithms is investigated in the following paragraphs. The dynamic characteristics obtained from the modal analysis using sub-microstrain data, that are processed with the peak-shift algorithms, are compared with the corresponding characteristics obtained from the FEM and from the modal analysis using accelerometer data only. Furthermore, the performance of the FPC and the MDA algorithms is experimentally investigated through an in-depth comparison of the modal characteristics obtained from the two algorithms. The basic working principles of both algorithms are described in the following sections.

Table 2
<table>
<thead>
<tr>
<th>Mode</th>
<th>1 (B-L)</th>
<th>2 (T)</th>
<th>3 (B-L)</th>
<th>4 (B)</th>
<th>5 (T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>f (Hz)</td>
<td>18.58</td>
<td>30.86</td>
<td>53.77</td>
<td>60.53</td>
<td>72.03</td>
</tr>
<tr>
<td>Mode</td>
<td>6 (B-L)</td>
<td>7 (T)</td>
<td>8 (B)</td>
<td>9 (B-L)</td>
<td>10 (T)</td>
</tr>
<tr>
<td>f (Hz)</td>
<td>108.73</td>
<td>128.44</td>
<td>171.89</td>
<td>183.26</td>
<td>202.36</td>
</tr>
</tbody>
</table>

Table 3
<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Maximum model order</th>
<th>Hankel's block rows i</th>
</tr>
</thead>
<tbody>
<tr>
<td>SSI-cov</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>CSI</td>
<td>70</td>
<td>70</td>
</tr>
</tbody>
</table>
The MDA searches for the wavelength that corresponds to the maximum power in the reflected spectrum. It is a pure peak detection algorithm which means that it does not take into account the shape of the spectrum. A proposed method to compute the wavelength with the maximum reflectivity is given in [21], where the following equation applies:

$$\lambda_{\text{max}} = \arg \max_{\lambda} \{pR(\lambda)\}$$

(1)

where $\lambda$ is the wavelength and $pR(\lambda)$ indicates the spectrum obtained with a $p$ point quadratic interpolation around the peak wavelength of the original reflection spectrum $R(\lambda)$ (Fig. 9). In this paper, it is assumed that $p=3$, since the interrogator that was used in the experiments also resolves the peaks with 2–3 points. By adding more quadratic points, the SNR is decreased and consequently the accuracy of the algorithm is significantly reduced. Another major disadvantage of the MDA is its sensitivity to the peak-locking effect which results in an underestimation of the actual strain level [21].

Table 4

<table>
<thead>
<tr>
<th>Mode</th>
<th>SSI-cov</th>
<th></th>
<th></th>
<th>CSI</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\sigma_f$ (Hz)</td>
<td>$\xi$ (%)</td>
<td>$\sigma_\xi$ (Hz)</td>
<td>$f$ (Hz)</td>
</tr>
<tr>
<td>(T) 2</td>
<td>30.94</td>
<td>0.04</td>
<td>1.00</td>
<td>0.16</td>
<td>30.77</td>
</tr>
<tr>
<td>(B) 4</td>
<td>60.45</td>
<td>0.04</td>
<td>1.16</td>
<td>0.13</td>
<td>60.71</td>
</tr>
<tr>
<td>(T) 5</td>
<td>71.17</td>
<td>0.11</td>
<td>0.44</td>
<td>0.11</td>
<td>70.97</td>
</tr>
<tr>
<td>(T) 7</td>
<td>127.54</td>
<td>0.10</td>
<td>0.06</td>
<td>0.15</td>
<td>127.62</td>
</tr>
<tr>
<td>(B) 8</td>
<td>173.35</td>
<td>0.04</td>
<td>0.31</td>
<td>0.02</td>
<td>173.79</td>
</tr>
<tr>
<td>(T) 10</td>
<td>203.73</td>
<td>0.04</td>
<td>0.07</td>
<td>0.06</td>
<td>203.69</td>
</tr>
</tbody>
</table>

Fig. 6. Unit normalized mode shapes of the beam, as obtained from the SSI-cov identification.
6.2. Fast phase-correlation (FPC)

The FPC algorithm determines the wavelength shift $\Delta \lambda$ from the phase shift between the undistributed or reference FBG spectrum $R(\lambda)$ and the perturbed spectrum $R'(\lambda) = R(\lambda + \Delta \lambda)$. One of the advantages of the FPC is that it takes into account the

![Fig. 7. Mass normalized mode shapes of the beam, obtained from the CSI identification and the FEM. The top subplot corresponds to the beam side with positive $y$-coordinates and the bottom subplot to the other side (Fig. 5). With continuous line (blue) are plotted the CSI mode shapes while with dashed line (red) the FEM ones. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)](image)

(a) Mode 2 (T) - 30.77 Hz
(b) Mode 4 (B) - 60.71 Hz
(c) Mode 5 (T) - 70.97 Hz
(d) Mode 7 (T) - 127.62 Hz
(e) Mode 8 (B) - 173.79 Hz
(f) Mode 10 (T) - 203.69 Hz
shape of the spectrum instead of only tracking the peak-shifts like the conventional algorithms, increasing in that way its accuracy and precision. Furthermore, it is less sensitive to the peak-locking effect while it is less sensitive to the noise than the MDA [20]. The value of $\Delta \lambda$ is computed by means of the following equation:

$$\Delta \lambda = \text{median}_{2 \leq k \leq M} \left( \angle \mathcal{R}(k) - \angle \mathcal{R}(k) \right) \frac{N \delta \lambda}{2\pi}$$

where $\mathcal{R}(k)$ and $\mathcal{R}(k)$ are the Fourier transforms of $R(\lambda)$ and $R'(\lambda) = R(\lambda + \Delta \lambda)$, respectively, $k$ is the generic Fourier spectral line, $M$ is the maximum number of Fourier spectral lines considered in the analysis, the symbol $\angle$ indicates the phase of the complex number and $N$ is the number of samples used for each spectrum. The number of samples $N$ depends on the wavelength scanning range $\lambda_{\text{max}} - \lambda_{\text{min}}$ and the wavelength resolution $\delta \lambda$ of the acquisition system:

$$N = \frac{\lambda_{\text{max}} - \lambda_{\text{min}}}{\delta \lambda}$$

It must be noted that one normally chooses $M=N$. However, the value of $M$ can be much lower than $N$ without considerably affecting the performance of the algorithm, since only the first few frequency lines of $R$ and $R'$ contain energy. Such an energy distribution is due to the shape of both spectra $R$ and $R'$, which can be approximated by sinc functions [20,21] (Fig. 10). For the current experiment, $M=9$ was considered as a good trade-off between execution speed and algorithm accuracy and precision.

The working principle of the FPC algorithm lays on the Fourier Transform property of a signal that a wavelength shift creates only a phase shift of it while it does not change its amplitude. To demonstrate this Fourier property, a sinc function is considered (Fig. 10) as the shape of the spectrum, with a wavelength of $\lambda=1540$ nm. Wavelength shifts of $\Delta \lambda = \lambda/2000$ nm, $\Delta \lambda = \lambda/3000$ nm and $\Delta \lambda = \lambda/4000$ nm are applied to the signal, following the Fourier Transform property:

$$\mathcal{F} \left( g(\lambda - \Delta \lambda) \right) = e^{-j2\pi f \Delta \lambda} \mathcal{F}(f)$$

where $g(\lambda)$ is the function representing the shape of the spectrum and $\Delta \lambda$ the wavelength shift. In Fig. 11, the amplitude and the
phase of the sinc signal are plotted, for four different wavelength shifts. It can be seen that the wavelength shifts do not alter the amplitude of the signal (all four amplitude graphs coincide in Fig. 11a) while they create phase shifts, which are used by the FPC algorithm in order to determine the wavelength shift according to Eq. (2).

7. Fiber-optic Bragg Grating (FBG) strain data modal analysis

In this section, the results of an output-only modal analysis, using strain data from the FBG sensors, are presented. The maximum model order and the half number of Hankel’s block rows, \( i \), were selected as described in Section 5 and are both equal to 150.

7.1. FPC–MDA raw data comparison

Both peak-shift algorithms were used for the processing of the raw wavelength data. In order to understand the differences between the two algorithms, the influence that they have on the raw data and consequently on the results that are obtained from the modal analysis are investigated. For this purpose, the RMS strain values for four sensors and for three tests of different excitation force amplitude are presented in Tables 5 and 6. Furthermore, the Power Spectral Density (PSD) of one sensor and for two of the tests are presented in Figs. 12 and 13 for comparison purposes. The PSD is calculated with Welch’s method [31], which involves sectioning the record, taking modified periodograms of these sections, and averaging these modified periodograms. The length of the sections was defined at 1024 points with an overlap between the sections of 66%, as the optimum trade-off values for obtaining clear peaks in the PSD.

It can be concluded from the RMS strain values and also from the amplitude of the peaks of resonance at the PSDs that the strain amplitude of the FPC treated data is consistently higher than the MDA. This is probably caused by the underestimation of the strain

Fig. 10. Shape of a sinc function spectrum.

Fig. 11. (a) Amplitude and (b) phase of a sinc wavelength: before the implementation of a wavelength shift (solid line) and for three wavelength shifts equal to \( \Delta \lambda = \lambda/4000 \) nm (dashed line), \( \Delta \lambda = \lambda/3000 \) nm (dotted line) and \( \Delta \lambda = \lambda/2000 \) nm (dash-dotted line).
7.2. Eigenfrequencies

The modal analyses that were performed for all the available test data indicate that the number of identified modes in the bandwidth of interest depends on two factors: the peak-shift algorithm that is used and the amplitude of the input force. In Table 7, the modes identified with SSI-cov from FPC and MDA data of test B are summarized. A significant observation is that the first lateral bending mode was identified from the FPC, while the measuring setup was designed to pick up only vertical and torsional modes (Section 2). This is probably due to a small deflection angle between the axis of the shaker and the vertical axis of the beam that activated this mode. All modes were selected from stabilization diagrams and have MPC values higher than 0.93, indicating almost purely real modes. The calculated standard deviation of the eigenfrequency values, as obtained from the SSI-cov identification, is also presented. Furthermore, the FEM eigenfrequencies (Table 2) are provided for comparison purposes.

The differences in the eigenfrequency values between FPC and MDA are minor and lower than 1.0%. The same comment applies when they are compared with the ones obtained from the conventional modal analysis (Section 5). Moreover, the standard deviation is in all cases smaller than 0.2 Hz, indicating a high identification accuracy. When the values of Table 7 are compared with the ones computed with the FEM, a good match is observed with the relative difference for the various modes to be less than ±1.5%. Overall, it can be concluded that the eigenfrequency values are well identified from the experimental strain data.

However, the identification capability of the MDA becomes lower for tests with lower force excitation. For example, Table 8 lists the eigenfrequencies obtained with both algorithms for test C. The performance of the FPC for test C, where the force excitation was very low, is better than the MDA’s, as in the range of [0:200] Hz, one mode is identified by the MDA for this low excitation test.

7.3. Modal strains

The modal strain shapes of the beam were also identified and are presented in Fig. 14, where they are also compared with the numerical modal strain shapes. The experimentally obtained modal strains are normalized to unit strain while the numerically obtained modes are mass normalized. A least-squares fit was therefore applied for the determination of the scale factor c that links the experimental unit normalized modal strains with the FEM mass normalized modal strains. The re-scaled modal strain shapes of the first six commonly identified modes from FPC and MDA treated data of test B are presented in Fig. 14, with respect to the FEM modal strains.

A significant observation is that torsional modes were identified from longitudinal strain sensors (Figs. 14a, c, d and f). The presence of the steel plates at the ends of the beam restrained the warping of the cross section at these locations [32], resulting in longitudinal stresses and strains which were picked up by the FBG strain sensors. The modal strains of the torsional modes vary in accordance with the effect of restrained warping [32], meaning that they reach their peak values where the warping is restrained e.g. the beams ends. There is also a link with the second derivative of the angle of twist, as can be derived from Fig. 7, and the longitudinal strains: zero second derivatives correspond to zero longitudinal strains as in the case of the middle of the beam for torsional modes 2 and 7. For the bending modal strains, the maximum values appear at the locations of maximum curvature, e.g. the middle of the beam for the fourth mode (Fig. 14b), where the modal curvatures are also maximized (Fig. 7b).

<table>
<thead>
<tr>
<th>Test #</th>
<th>Force (N)</th>
<th>FBG 2 (με)</th>
<th>FBG 5 (με)</th>
<th>FBG 11 (με)</th>
<th>FBG 13 (με)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>13.46</td>
<td>1.706</td>
<td>1.814</td>
<td>1.835</td>
<td>1.870</td>
</tr>
<tr>
<td>B</td>
<td>2.37</td>
<td>0.390</td>
<td>0.474</td>
<td>0.513</td>
<td>0.554</td>
</tr>
<tr>
<td>C</td>
<td>0.42</td>
<td>0.151</td>
<td>0.257</td>
<td>0.373</td>
<td>0.378</td>
</tr>
</tbody>
</table>

Table 5
RMS force and strain values for Test A, B and C, processed with FPC algorithm.

<table>
<thead>
<tr>
<th>Test #</th>
<th>Force (N)</th>
<th>FBG 2 (με)</th>
<th>FBG 5 (με)</th>
<th>FBG 11 (με)</th>
<th>FBG 13 (με)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>13.46</td>
<td>0.962</td>
<td>0.922</td>
<td>0.976</td>
<td>0.977</td>
</tr>
<tr>
<td>B</td>
<td>2.37</td>
<td>0.216</td>
<td>0.287</td>
<td>0.317</td>
<td>0.359</td>
</tr>
<tr>
<td>C</td>
<td>0.42</td>
<td>0.113</td>
<td>0.206</td>
<td>0.261</td>
<td>0.280</td>
</tr>
</tbody>
</table>

Table 6
RMS force and strain values for Test A, B and C, processed with MDA algorithm.
Fig. 14 indicates a better performance of the FPC algorithm in modal strain identification than the MDA. More specifically, the FPC modal strain shapes (blue dashed line) are smoother and pose smaller relative amplitude differences with the FEM than the MDA (red dotted line). The better performance of the FPC is further supported by the implementation of the MAC, as a measure of consistency (degree of linearity) between the estimates of the modal vectors. The MAC values of the FPC with the FEM, when the same modal shapes are compared, are over 0.96 while of the MDA with the FEM over 0.91, giving a better consistency of the FPC with a validated numerical model (Section 5).

Furthermore, the identified unit normalized modal strain shapes with their 95% confidence interval, which represents the variance error of the output-only identification with SSI-cov [25], are presented in Fig. 15 for both FPC and MDA. The FPC strains are denoted with blue while the MDA with red color. The top subplots refer to the top flange of the beam while the bottom to the bottom one. It can be easily observed that the range of the 95% confidence interval for the MDA is larger than the FPC, which translates into a lower identification confidence.

8. Combined data modal analysis

In order to obtain mass normalized modal strains a combined modal analysis using both acceleration and strain data was conducted. The synchronization of the acceleration and the strain acquisition systems was performed as described in Section 2.3. A combined modal analysis offers the benefit of mass normalized modal strains since the direction and the location of the input force coincided with one of the accelerometers. The system identification was conducted also with the SSI-cov technique so that the variance error of the identification could be quantified.

In Table 9, the selected values for maximum model order and the half number of Hankel's block rows \( i \) are summarized. Since there are more input DOFs for the combined analysis than for the single analyses, the computational time increases significantly, forcing the user to select lower values for these quantities than the single data analyses which, however, still meet the minimum requirements.

The identified mode shapes and eigenfrequencies from the combined modal analysis are well matching the ones obtained from the single data modal analyses. The differences are minor and lower than 1% for the eigenfrequency values and 5% for the modal
displacements amplitude. Therefore, these values are not presented here. In contrast, the mass normalized modal strains are extensively discussed.

8.1. Modal strains

The experimentally obtained mass normalized modal strains from the CSI identification versus the numerically obtained ones from the numerical model are presented in Fig. 16. The modal strains from both FPC and MDA peak-shift algorithms and for test B are shown. The FPC mass normalized modal strain shapes (blue dashed line) are smoother and pose significantly smaller relative amplitude differences with the FEM than the MDA (red dotted line). The underestimation of the actual strain level from the MDA, due to the peak locking effect, is apparent. The underestimation can be up to 160% off the actual value which translates in failing to identify the actual strain amplitude. On the contrary the estimation of the strain amplitude by the FPC is quite accurate and the differences are less than 20% with the validated FEM. The aforementioned indicates a much better performance of the FPC algorithm in modal strain identification than the MDA.

Furthermore, the modal strain shapes with their 95% confidence interval, for both FPC and MDA, as they are calculated from the SSI-cov, are presented in Fig. 17. It can be easily observed that the range of the 95% confidence interval for the MDA is larger than the FPC, which translates into a lower identification confidence for the MDA. The range of the 95% confidence interval is larger in the combined data than the FBG data analysis since lower system order and number of Hankel’s block rows have been selected to keep the computational time in acceptable levels due to the larger number of inputs. This contributes to lower identification accuracy which mainly influences the MDA modal strains.

9. Conclusions

This paper focuses on modal strain identification from sub-microstrain dynamic measurements. A novel peak-shift algorithm, the Fast Phase Correlation algorithm or FPC, has been applied to increase accuracy and precision. Its performance was investigated and the obtained modal characteristics were compared with the corresponding ones from a conventional peak-shift algorithm, the Maximum Detection Algorithm or MDA.

For the low excitation force amplitude tests, where the signal to noise ratio (SNR) was still reasonably high, the FPC algorithm was able to identify all the modes within the frequency interval of interest. On the contrary, the MDA could usually identify only the bending modes present in the same range. Furthermore, the modal strain identification was more accurate with the FPC. The amplitude of the mass normalized modal strains is well identified by the FPC while it is highly underestimated by the MDA. In terms of eigenfrequency, both FPC and MDA identified the same values with negligible differences which were confirmed also by the

### Table 7

<table>
<thead>
<tr>
<th>Mode</th>
<th>FPC</th>
<th>MDA</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\sigma_f$ (Hz)</td>
<td>$\xi$ (%)</td>
</tr>
<tr>
<td>(T) 2</td>
<td>31.04</td>
<td>0.08</td>
<td>1.63</td>
</tr>
<tr>
<td>(B-L) 3</td>
<td>53.82</td>
<td>0.03</td>
<td>0.08</td>
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<td>(B) 4</td>
<td>60.30</td>
<td>0.14</td>
<td>1.14</td>
</tr>
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<td>(T) 5</td>
<td>71.18</td>
<td>0.09</td>
<td>0.26</td>
</tr>
<tr>
<td>(T) 7</td>
<td>127.49</td>
<td>0.03</td>
<td>0.06</td>
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<tr>
<td>(B) 8</td>
<td>173.54</td>
<td>0.07</td>
<td>0.27</td>
</tr>
<tr>
<td>(T) 10</td>
<td>203.73</td>
<td>0.01</td>
<td>0.07</td>
</tr>
</tbody>
</table>

### Table 8

<table>
<thead>
<tr>
<th>Mode</th>
<th>FPC</th>
<th>MDA</th>
<th>FEM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f$ (Hz)</td>
<td>$\sigma_f$ (Hz)</td>
<td>$\xi$ (%)</td>
</tr>
<tr>
<td>(T) 2</td>
<td>30.86</td>
<td>0.05</td>
<td>1.00</td>
</tr>
<tr>
<td>(B) 4</td>
<td>60.52</td>
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<td>1.00</td>
</tr>
<tr>
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<td>70.98</td>
<td>0.08</td>
<td>0.21</td>
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<tr>
<td>(T) 7</td>
<td>127.51</td>
<td>0.07</td>
<td>0.15</td>
</tr>
<tr>
<td>(B) 8</td>
<td>173.27</td>
<td>0.13</td>
<td>0.24</td>
</tr>
<tr>
<td>(T) 10</td>
<td>203.49</td>
<td>0.07</td>
<td>0.05</td>
</tr>
</tbody>
</table>
The accuracy in the identification of the structural characteristics from FBG strain data, processed with FPC peak-shift algorithm, is at least of the same magnitude with the characteristics identified from the conventional accelerometer data modal analysis. It is a remarkable achievement that torsional and lateral bending modes were identified from strains measured in the longitudinal axis of the beam. Therefore, the FBG strain sensors can serve as a valuable, accurate and cost effective option for the vibration-based structural health monitoring (VBSHM) where the accuracy in modal strain identification is of major importance for locating early stage damage in structures. These new findings make it possible to obtain modal strain shapes from sub-microstrain ambient or

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**Fig. 14.** Modal strain shapes of the beam, obtained from SSI-cov identification for test B and the FEM. Dashed line (blue): FPC modal strains, dotted line (red): MDA modal strains, dash-dotted line (green): FEM modal strains. Top subplots correspond to top flange fiber while bottom to the bottom one. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
operational strain data, which is an important step in the practical modal testing and the VBSHM of civil infrastructure.

Fig. 15. Unit normalized modal strains (continuous line) and 95% confidence interval (dotted line), identified with SSI-cov for test B. Left subplots (blue): FPC modal strains, right subplots (red): MDA modal strains. Top subplots correspond to top flange fiber while bottom to the bottom one. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
Table 9
Selected parameters for SSI-cov and CSI - combined data analysis.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Maximum model order</th>
<th>Hankel's block rows i</th>
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<tr>
<td>SSI-cov</td>
<td>120</td>
<td>120</td>
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<tr>
<td>CSI</td>
<td>100</td>
<td>70</td>
</tr>
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</table>

Fig. 16. Mass normalized modal strain shapes of the beam, obtained from CSI identification for test B. Dashed line (blue): FPC modal strains, dotted line (red): MDA modal strains, dash-dotted line (green): FEM modal strains. Top subplots correspond to top flange fiber while bottom to the bottom one. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
Fig. 17. Modal strains (continuous line) and 95% confidence interval (dotted line), identified with SSI-cov for test B. Left subplots (blue): FPC modal strains, right subplots (red): MDA modal strains. Top subplots correspond to top flange fiber while bottom to the bottom one. The modal strains were normalized with the accelerations to unit modal displacement. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)
Acknowledgments

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References