Numerical modeling of femtosecond laser inscribed IR gratings in photonic crystal fibers

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Abstract: During grating inscription in photonic crystal fibers (PCFs) the intensity of the inscribing laser beam is non-uniformly distributed over the core region due to the interaction with the air holes in the fiber’s microstructure. In this paper we model and study the non-uniformity of the index modification and its influence on the grating reflection spectra, taking into account the non-linear nature of the index change. For femtosecond laser inscription pulses at 800 nm, we show that the intensity redistribution in the PCF core region can result in Type II index changes even if the peak intensity of the incident beam is well below the corresponding threshold. Our coupled mode analysis reveals that the non-uniform nature of the index change can seriously affect the reflectivity of the grating due to a limited overlap of the guided mode with the transverse index modulation profile for almost all angular orientations of the PCFs with respect to the inscription beam. We also evaluate the influence of PCF tapering and we found that for the considered PCF a significant increase in the induced index change and reflectivity is observed only for taper diameters below 40 μm.

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References and links

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1. Introduction

The development of photonic crystal fiber (PCF) technology can be considered as a major achievement in the field of photonics. PCFs offer unprecedented design flexibility and can possess unique waveguiding features. Solid core PCFs commonly consist of a glass core surrounded by a cladding formed with a periodically arranged lattice of wavelength scale air holes that run along the entire length of the fiber [1].

The fabrication of fiber Bragg gratings (FBGs) in PCFs is a subject of intensive research [2]. A FBG is a periodic variation of the refractive index induced in the core of a single mode optical fiber, which acts as wavelength selective mirror [3]. Applications of PCF based FBGs

can be found in the fields of optical communications, fiber lasers and optical sensors [4]. The benefits of using PCFs instead of conventional fibers for sensing applications have been discussed in [5,6], for example. Temperature, strain, pressure and even shear stress sensors based on FBGs inscribed in PCFs have been reported [6–9]. For specific applications such as pressure sensing in the energy industry, one would ideally require gratings that remain stable at high temperature (e.g. above 500 °C) [6]. Such gratings in PCFs would also be of interest for developing high power all-fiber lasers [10].

High temperature resistant Type II gratings can be considered for such high-temperature applications. They are usually inscribed as a result of a highly non-linear multi-photon absorption process [11–14] using a femtosecond laser source. While they have been successfully manufactured in conventional optical fibers using near IR femtosecond laser sources, femtosecond fabrication of gratings in PCFs is not straightforward [2,11]. The presence of air holes in the PCF cladding region impedes optical power delivery to the fiber core which can prohibit efficient grating inscription [2,12,15–19]. Procedures such as filling the air holes with index matching liquid or fiber tapering have been proposed to mitigate the detrimental influence of the holey cladding on the grating writing [20–23]. A new technique for obtaining high temperature stable relief Bragg grating writing using a wet etching technique in the air holes of a PCF was also proposed recently [24]. New PCF designs allowing efficient grating inscription could also provide a solution to this problem, especially for sensing applications that impose less severe restrictions on the PCF dispersion properties and/or losses [25,26].

Yet femtosecond grating inscription with IR pulses in PCFs is still considered to be problematic due to highly non-linear nature of the index change process [2]. Only a few experimental reports discussed the importance of the PCF orientation and the detrimental influence of the air holed cladding were emphasized [21,22]. Several research efforts were already conducted to gain a better understanding of the influence of the fiber’s microstructure on the efficiency of fiber Bragg grating writing in PCFs [2]. In most numerical studies the influence of the holey cladding on the amount of light reaching the PCF core was studied by obtaining the intensity distributions in the fiber’s cross-section [2,12,15–17]. Previously, we have also used such an approach to study the influence of lattice parameters and of the angular orientation of a hexagonal lattice PCFs with respect to the inscribing beam on the transverse coupling efficiency [18,19,27,28]. However such simulations can only give partial insights into the influence of the microstructure because they do not account for the effect of the irregularities of the induced index changes on the reflection strength of the grating.

Here we present an extension to our previous approach and we propose a new methodology to study the influence of air holes in PCF cladding region on the resulting grating efficiency for femtosecond laser based inscription methods at 800 nm. To do so we model the intensity distribution, we estimate the resulting induced index change in the PCF core taking the non-linear nature of light-matter interaction into account and finally we also calculate the reflection strength of the inscribed FBG. More specifically, first we use the transverse intensity distributions derived from FDTD simulations and empirical data from literature for IR femtosecond gratings to model the distribution of the refractive index modification in the PCF core region. As the intensity distribution is not uniform in the PCF core region, the resulting index change is not uniform either. We then study the influence of this non-uniform refractive index change on the resulting grating spectra using coupled mode theory. For that purpose we extend the approach already used for modeling the reflection spectra of point-by-point gratings in conventional fibers [29]. Finally we also study the influence of the PCF orientation and of fiber tapering on the resulting grating reflectivity for two particular PCFs.

Our paper is structured as follows. In Section 2 we present our model for calculating the induced index change in the PCF core region based multi-photon absorption and we deal with the influence of the PCF orientation on the average induced index change. In Section 3 we estimate the influence of the non-uniform nature of the index change over PCF cross section on the resulting grating reflectivity using couple mode theory. In Section 4 we simulate the
influence of PCF tapering on the average induced index change and grating reflectivity. We close our paper and conclude with Section 5.

2. Modeling non-linear refractive index modulations induced by high intensity femtosecond laser pulses in a PCF core

In this Section, we simulate the refractive index modification in the PCF core region based on intensity distributions obtained with FDTD simulations and on empirical data from literature for 800 nm femtosecond pulses. We note upfront that although exact values of the index change will depend on the details of the actual inscription setup, such as the focusing geometry, laser parameters, illumination time, etc., and on the exact composition of the fiber glass, our approach should offer reasonable understanding of the influence of the PCF’s microstructure on the induced index change distribution.

2.1 Grating inscription set-up and fibers under test

We consider 800 nm femtosecond grating inscription with an interferometric setup. This means that grating inscription is carried out with a side-illumination that consists in transversely exposing the optical fiber core to an interference pattern. The resulting index modulation then lies parallel with the optical fiber axis, with grating planes that are perpendicular to that axis.

The outline of the interference scheme considered in our paper is illustrated in Fig. 1(a). Purely two beam interference configuration applies – to some extent – to femtosecond phase mask inscription as well, due to the so-called walk-off effect of the different diffracted orders, which results in pure two beam interference of only the +1 and −1 diffracted orders at a certain distance from the phase mask (see [11,30] for more details).

Fig. 1. Illustration of (a) two interfering beams that transversely propagate to the PCF core region, (b) and (c) cross sections of the considered PCFs with wave vector decomposition of the transverse beam into in- and out-of-plane components.

As a reference setup throughout our study we considered that used by Smelser et al. in [13], where gratings were inscribed in conventional step-index fiber using 800nm 125 fs pulses from a Ti:sapphire laser system. The radius of the exit beam was measured as 3.2 mm, while a cylindrical lens with a focal distance of 30 mm with a resulting Gaussian beam waist radius of 2.4 μm was used to focus the pulses on a fiber with a repetition rate up to 1 kHz. In that work a phase mask with a period of 3.2 μm was used to form the interference pattern. In addition the beam was scanned along the direction parallel to the phase mask groves in order to increase the overlap of the induced index change region with the fiber core. The induced refractive index change in a particular point in the core region is thus the result of thousands of pulses with various peak intensity values resulting from the beam scanning.

The fibers under test in our work are two hexagonal lattice PCF geometries, which are depicted in Fig. 1(b) and 1(c). The first PCF structure, denoted as PCF-1, has relatively small lattice parameters (air hole pitch of 2 μm and air hole diameter of 1.8 μm) as well as a small core diameter. The dimension of the core region is comparable to the waist of the writing beam. The second PCF depicted in Fig. 1(c) is the commercially available ESM-12-01 PCF [31], which we will denote as PCF-2. The air hole pitch of this PCF is 8 μm and the air hole
diameter measures 3.68 μm. Both PCFs are single mode at 1550 nm. Due to the differences in their cross-sectional geometry, the PCFs exhibit different transverse coupling properties. These differences in the cross-sectional geometries will help to illustrate their effect on the transverse coupling efficiency, on the associated grating writing process and on the eventual Bragg grating properties.

2.2 The non-linear refractive index change in silica fibers

We first recall the principles of femtosecond pulse induced refractive index modification in silica. The photon energy at 800 nm is around 1.55 eV, while the bandgap of pure silica is around 9.3 eV and 7.1 for 5% Ge doped silica [14]. At 800 nm at least five photons are necessary to induce a refractive index change in the PCF core, as demonstrated in [13]. Note that in a work of the same authors [32] a 6 photon absorption mechanism was also considered as a possible process leading to refractive index modifications. Yet working with either a 5 or 6 photon process would not have a dramatic influence on the conclusions drawn below and hence, in the remainder of this paper, we will assume that this five photon process is responsible for the induced refractive index change.

As stated in literature [13,14], depending on the peak intensity of the pulse, three different index modification mechanisms can occur: above approximately $2 \times 10^{13}$ W/cm$^2$ the refractive index change is generally attributed to reversible color center formation (associated with Type I-IR gratings), above around $4.6 \times 10^{13}$ W/cm$^2$ irreversible birefringent modification takes place (associated with Type II-IR gratings) and around $30 \times 10^{13}$ W/cm$^2$ nanovoid formation occurs due to strong electron plasma generation and resulting shock wave propagation (involved in the formation of so-called point-by-point gratings) [33,34].

For IR femtosecond interferometric grating writing, the first two mechanisms are usually involved. In [13], experimental demonstrations of both Type I-IR and II-IR grating inscription in Ge-doped SMF-28 fiber were reported and a 5-photon absorption mechanism was shown to be responsible for the refractive index changes.

To obtain an idea about the eventual refractive index profile in the PCF core region we propose the following simplified model, with magnitudes of the refractive index modifications taken from [13]. The values that are eventually used for our calculation are indicated between parentheses.

- $I < 2 \times 10^{13}$ W/cm$^2$: no index change ($\Delta n = 0$);
- $2 \times 10^{13}$ W/cm$^2 < I < 4.6 \times 10^{13}$ W/cm$^2$: reversible color center formation mechanism is responsible for the refractive index change through a five photon absorption process, which can be calculated with:

$$\Delta n = C \cdot I^5$$

where $C$ is a constant coefficient (refractive index change in the range of $\Delta n = 0.1 \times 10^{-4}$ to $6 \times 10^{-4}$ and $C = 3.23 \times 10^{-3}$);
- $I > 4.6 \times 10^{13}$ W/cm$^2$: irreversible Type II index change takes place, which we consider to have a fixed value. According to the literature this index change is of the order of $10^{-3}$ (constant refractive index modification $\Delta n = 10^{-3}$).

Figure 2 shows the refractive index modulation as a function of the intensity according to the model presented above. The experimental values taken from [13] are graphed in Fig. 2, where gratings with input pulse energy from 850 μJ to 1200 μJ were inscribed corresponding to the peak intensities from around $I = 3 \times 10^{13}$ W/cm$^2$ to $I = 4.3 \times 10^{13}$ W/cm$^2$. 
Fig. 2. Proposed simplified model for the refractive index change as a function of the optical intensity at 800 nm.

Additionally we assumed that above $I = 4.6 \times 10^{13}$ W/cm$^2$ the refractive index is constant, in accordance with the Type II nature of the index change. The refractive index modification beyond this range of intensities is still a matter of investigation [35]. The non-linear nature of light matter interaction, self-focusing of the beam and structural damage of the host material make it very complex to accurately model the eventual values of the index change. Nevertheless saturation is expected in this region, which we included in our model with a maximum value of $1 \times 10^{-3}$.

2.3 Modeling transverse coupling involving beam scanning for grating inscription and calculating the distribution of maximal intensities in the PCF core region

As already mentioned, in order to increase the overlap with the fiber core one can scan the beam in a direction parallel to the phase mask groves. In [13] the inscription beam was vertically scanned along the Y axis with a piezo-actuated translation stage over a range of $\pm$ 10 $\mu$m, as the waist radius of the beam (2.4 $\mu$m) is smaller than the core dimension of the fiber used (around 8 $\mu$m). We also took this beam scanning procedure into account in our simulations. To do so, we performed a transverse propagation simulation for each position of the scanning beam.

As illustrated in Fig. 1(a) the problem of transverse coupling for interferometric inscription methods is three-dimensional in general, as the writing beams are impinging on the PCF under some angle $\alpha$ relative to the cross-section plane of the fiber. Figure 1(b) shows the wave vector decomposition into in-plane ($k_{xy}$) and out-of-plane ($k_z$) components. The $k_z$ component is directed along the fiber and is responsible for the formation of the interference pattern. It does not interact with the microstructure, while the $k_{xy}$ component lies in the plane of the PCF cross-section and thus interacts with air holes.

Therefore the problem of transverse coupling in a PCF can be reduced to a two-dimensional problem by simulating only the wave vector component $k_{xy}$. The effective wavelength for the input beam in 2D simulations should then be specified as:

$$\lambda_{xy} = \frac{\lambda}{\cos(\alpha)}$$

(2)

where $\lambda_{xy}$ corresponds to the in-plane wave vector component $k_{xy}$.

2D simulations were already reported several times to deal with similar problems [12,15,16,18,19]. We emphasize here that to model phase mask inscription the effective wavelength should have been used in these papers, as discussed below.

For our reference setup [13], a simple calculation shows that for incident light with a wavelength of 800 nm the angle between the first $+1$ and $-1$ diffracted orders from a phase
mask with a period \( \Lambda = 3.213 \, \mu m \) relative to the direction perpendicular to the fiber is 14.43°. Hence, according to Eq. (2) the effective wavelength \( \lambda_{eff} \) for our 2D transverse coupling simulations is 826 nm.

The input pulse was further specified as a Gaussian beam with a 2.4 \( \mu m \) waist and with a peak intensity of \( I = 3 \times 10^{13} \, W/cm^2 \). Here we performed separate transverse propagation simulations for each position of the scanning beam (see Fig. 3(a)) using the commercially available Lumerical FDTD software [36]; a more detailed description of the simulation approach has been reported earlier [19].

Fig. 3. (a) Outline of the multiple beam scanning modeling approach for PCFs and process for obtaining the distribution of maximal intensities in the fiber core region for (b) a conventional step-index fiber and (c) a photonic crystal fiber.

For PCF-1, beam scanning in a range \( \pm 5 \, \mu m \) with a step of 0.5 \( \mu m \) was found to be sufficient to cover the entire core region due to its small size around 5 \( \mu m \). We thus obtained 21 intensity distribution patterns each corresponding to a certain position of the inscription beam. For PCF-2, scanning in a range of \( \pm 10 \, \mu m \) with a step of 0.5 \( \mu m \) resulted in 41 patterns. In the simulations the propagation through the outer cladding was also taken into account. More detailed explanations of the simulation approach can be found in earlier work [19].

In order to reconstruct the refractive index modification profile from those multiple simulations we used so called distribution of maximal intensity. The procedure for obtaining those distributions for conventional step index fiber and PCF is illustrated in Fig. 3.

As we see in Fig. 3(b) for conventional fiber the intensity in the certain regions of the core is not always maximal and depends on the position of the scanning beam. So each of the pulses induces a certain index change in a certain part of the core and in the end a uniform refractive index change is obtained owing to the scanning across the entire fiber width. An important aspect here is that every point of the core was illuminated with the peak intensity of the writing beam. This can be seen from the distributions of maximal intensity that were obtained by taking maximum values of all the 21 simulations as illustrated in Fig. 3(b). The same procedure was used for PCF-1 as shown in Fig. 3(c), where more complex distributions are observed due to the interaction of the writing beam with the holey cladding. To some extent this approach is taking into account the accumulated influence of all the beam positions, as the values taken in the model are obtained under identical illumination conditions as in case of conventional fibers.

2.4 Refractive index modulation in the core region of the PCF

To reconstruct the refractive index modulation we combine the models described in Section 2.2 and 2.3.

The maximum intensity distribution modeled with the approach presented in Section 2.3 for PCF-1 is shown in Fig. 4(a) after illumination with a beam that is incident along the \( \Gamma \)M
direction of the hexagonal lattice, together with the resulting index change. The simulations used a peak intensity of the incident Gaussian beams \( I = 3\times10^{13} \text{ W/cm}^2 \), which would typically induce Type I refractive index modifications in conventional fibers [14]. The maximum intensity distribution for PCF-1 returns very low intensities for each of the scanning beam positions at many locations in the PCF core, and hence no high refractive index change can be expected in those places. However, in other parts of the core the intensity is twice as high compared to a conventional fiber as a result of the interaction with the air holes.

In Fig. 4(a) we can also observe that in some regions the intensity level is above the Type II modification threshold, which is interesting considering that the maximum intensity of the impinging beam \( 3\times10^{13} \text{ W/cm}^2 \) was below that threshold. Therefore, and as shown in Fig. 4(b), the intensity redistribution results in some regions with higher and even Type II index changes; whilst no index change is detected in other vast regions of the core as the intensity remains below Type I threshold. This result predicts that the temperature stability of the grating inscribed in PCF and conventional step-index fiber may be different, solely due to the intensity redistribution phenomenon observed in PCFs.

The non-uniform nature of the index change in the PCF core region modeled here is in good agreement with the results from literature. In particular the non-uniform index distributions in the PCF core region, such as depicted in Fig. 4(b), were also measured experimentally although with other femtosecond writing methods. In [37] such index change patterns as a result of point-by-point grating inscription were detected and good agreement with the modeling intensity distribution patterns were observed. More recently index changes in a PCF cross-section after phase mask FBG inscription at 266 nm were measured, again revealing the non-uniform nature of the index change in the core region [38].

We also calculated the averaged refractive index change \( \Delta n_{av} \) in the PCF-1 core and compared it to that obtained in conventional fiber in the same illumination conditions (see Fig. 3). The PCF core region is defined as a circle that touches the closest air holes. For a fiber with no air holes the average refractive index change is \( \Delta n_{av} = 0.74\times10^{-4} \), whilst for the PCF-1 this value is \( \Delta n_{av} = 0.78\times10^{-4} \). Although high intensities in PCF-1 are observed only in a limited region of the core, the average refractive index modulation in the PCF-1 is of the same order and even slightly higher than that in conventional fiber. This is a consequence of the energy redistribution and of the accompanying non-linear index change mechanism.

2.5 The influence of the PCF orientation on the non-linear index change

We simulated the maximum intensity distributions for PCF-1 and PCF-2 for different angular orientations of the microstructure with respect to the direction of the inscription beam – under identical illumination conditions – and we calculated the average of the induced refractive index modification in the core region. Due to the symmetry of the hexagonal lattice we have limited the simulations to angular orientations from 0° to 30°, with 1° increments. To obtain
one graph for a parameter as a function of the angular orientation and due to the use of the scanning beam method, we had to carry out a total of 651 simulations for PCF-1 and 1271 simulations for PCF-2.

Figure 5 shows the average induced refractive index change as a function of the PCF orientation for writing beam peak intensities of $I = 3 \times 10^{13}$ W/cm$^2$ and $I = 4.3 \times 10^{13}$ W/cm$^2$. PCF-1 has small features in the holey cladding (Fig. 5(a)) and returns much higher average refractive index values. For all orientations the induced refractive change is non-zero. We find a maximal average index change for an orientation at 30°, i.e. for inscription along the $\Gamma M$ direction of the hexagonal lattice. Note that this last result for $\Gamma M$ orientation is in good agreement with previous findings for a similar PCF geometry [19]. For a pulse with $I = 3 \times 10^{13}$ W/cm$^2$ the average of the induced index change reaches almost $0.8 \times 10^{-4}$, while for a pulse with $I = 4.3 \times 10^{13}$ W/cm$^2$ it reaches almost $2.1 \times 10^{-4}$.

PCF-2 returns much lower values of the average index change with a maximum refractive index modification around $0.9 \times 10^{-5}$. For a wide range of orientations (15° - 45°) no grating growth is expected due to poor transverse coupling. This result is in good agreement with earlier reports. The importance of the angular orientation inducing grating growth in this particular PCF under similar illumination conditions was evidenced experimentally in [21]. A quantitative comparison of our results with [21] is not straightforward however, due to the longer exposure times, the slightly different illumination conditions and the achieved saturation of the index change.

We also see that by increasing the writing pulse energy and hence the interference pattern peak intensity from $I = 3 \times 10^{13}$ W/cm$^2$ to $I = 4.3 \times 10^{13}$ W/cm$^2$, the refractive index modulation increases significantly. The latter effect stems from the highly non-linear 5-photon absorption process.

We conclude that the induced refractive index change values for PCF-1 and PCF-2 differ by more than one order of magnitude, which underlines the influence of the cross-sectional geometry of the PCF on grating writing. This also confirms that the efficiency of FBG inscription in PCFs can be significantly increased with dedicated PCF designs.

3. The influence of the non-uniform index modification in a PCF core on the fiber Bragg grating reflection

In this Section, we model the grating peak reflectivity based on the simulated transverse index distribution in PCF core. The procedure is schematically illustrated in Fig. 6, where the process of obtaining grating reflection for two cases of uniform and non-uniform index
modifications of the grating periods are shown. In Section 2.4 we have described a methodology to reconstruct the index profile and we used the averaged index change as a figure of merit to estimate the influence of the angular orientation on grating writing. However we cannot only use the values of the average refractive index change to estimate the grating reflection level, as it neglects the effect of the non-uniform index distribution.

The overlap of the guided mode with the non-uniform index profile needs to be taken into account when modeling grating reflection. To do so we applied the well-known coupled mode theory [39]. It was also recently used to study peculiarities of the reflection spectra of point-by-point gratings, for which similar non-uniform index modification were evidenced [29].

![Fig. 6](image)

Fig. 6. Schematic illustration of the simulation approach used to model the spectral response of a fiber Bragg grating in photonic crystal fiber.

We are interested in coupling forward and backward propagating fundamental modes by means of grating structures. The coupling coefficient for the fundamental mode is calculated as follows:

\[
\kappa = \frac{\omega}{4} \int \int \Delta \varepsilon(x, y) \bar{E}_i \cdot \bar{E}_f \, dx \, dy
\]

where \(\bar{E}_i\) is the normalized transverse field distribution of the PCF mode (see Fig. 6, top-left) and \(\Delta \varepsilon\) is the transverse distribution of the permittivity, which can be approximated as \(\Delta \varepsilon \equiv 2\Delta n\). The integration of the overlap integral is calculated over the PCF core region, which we defined as a circle that touches the closest air holes. Calculations show that for the PCFs considered here, more than 97% of the energy of the fundamental mode is concentrated in that region. The profile of the transverse index modification (such as shown in Fig. 4) is taken into account when calculating the overlap integral in Eq. (3). We assume that this transverse index modification is cosine modulated along the fiber length. A consistent grating length of \(L = 2 \, \text{mm}\) is used in the calculations.

The peak reflectivity of the grating can then be calculated with the following formula [39]:

\[
R = \tanh^2(\kappa L)
\]

Using the tools described above we can calculate the reflectivity of the first order fiber Bragg gratings around a 1550 nm wavelength using the data of the transverse index modulation for PCF-1 and PCF-2 modeled in the previous sections. The resulting dependence of the grating reflectivity on the angular orientation of the PCF is shown in Fig. 7. The lines in bold correspond to the reflectivity obtained with the non-uniform refractive index profile, whilst the dotted lines represent the reflectivity if non-uniformities of the transverse index...
modulation are neglected and the average values for the index change are used instead. The reflectivity values here are presented for the case of Gaussian beams with peak intensity of \( I = 4.3 \times 10^{13} \text{ W/cm}^2 \), which correspond to the red curves of the results of average index change presented in Fig. 5.

A first important observation is that the reflectivity calculated with the average value of the transverse index modification (dotted line) is always higher than that calculated with the non-uniform index change. This means that the non-uniform nature of the index change, as expected, has a detrimental influence on the resulting grating reflection level, although the index change can be higher than the average value and even of Type II at some locations in the core region.

For PCF-1 the reflection values are much higher than for PCF-2, which is in accordance with the results of the previous section. In general the curves correlate well with the results of the average index change presented in Fig. 5. For the given FBG configuration and at a 30° orientation a reflectivity as high as 0.27 is obtained for PCF-1 as can be seen from Fig. 7(a). For the average value of the index change the model predicts a value of 0.49.

The difference between the reflectivities calculated with the average values (dotted curve) and when irregularities are taken into account (bold line) reaches the largest value (a factor 4.2) for a PCF-1 orientation at 23° (37°). The smallest difference of a factor 1.8 is observed at a 30° orientation. Those differences stem from the overlap of the index change regions with the fundamental mode. When the index change in the PCF core region takes place at the edges where the intensity of the guided mode is low, poor coupling is achieved. Apparently for PCF-1 the best coupling conditions occur for FBGs inscribed at an orientation of 30° (TM direction), which also coincides with the highest value of the average index change.

For PCF-2 a very low reflectivity is calculated for the given model parameters, with a maximum of around 3.8x10^{-3}. The difference between the reflectivity calculated with the average values of the index change and that with the non-uniform index distribution is much larger than for PCF-1. Best coupling is observed at 0° (or when the grating is inscribed along the ΓK direction of the lattice), where the values differ with a factor 3. For comparison, at the second local maxima at 9° and 51° this difference is around 9 times. At these orientations the index change again took place at the edges of the core regions resulting in lower values of the overlap integral and coupling coefficient.

Hence, in case of PCFs we are dealing with the interplay of two opposite effects. On one hand the inhomogeneous intensity distribution in the core region increases the non-linear absorption and hence the induced refractive index change. On the other hand the same irregularity of the intensity distribution lowers the reflectivity level due to the limited overlap.
with the guided mode. As one could expect the location of the index change is important; if the induced index change occurs at the edges of the core region then the coupling coefficient is lower than when the index change happens in the central region of the core. We showed that for the PCFs considered here the non-uniform nature of the transverse index change can result in a 2- to 10-fold reflectivity drop.

From these results we can conclude that in most cases the non-uniform distribution of the transverse index modification in the PCF core region decreases the reflectivity of the grating. Furthermore, when estimating the average induced index change in the PCFs by measuring the reflectivity of the grating, the non-uniform nature of the index change should be taken into account. The induced index change of the inscribed grating in PCF can be higher, while the reflectivity is lowered due to the non-uniform nature of the index change.

4. Influence of PCF tapering on the resulting grating efficiency

Finally we study the influence of fiber tapering on the induced index change and resulting reflectivity of the grating. The use of PCF tapering to increase the grating reflectivity was already dealt with several times in literature [21–23]. Taper diameters varying from 30 µm to 55 µm for differing PCF microstructures were reported in those works. Moreover, for some specific PCFs tapering was considered as the only solution to mitigate the detrimental influence of the air holes [21,22].

We consider PCF-2 under the same illumination conditions as previously in the paper with inscription beam peak intensity $I = 4.3 \times 10^{13}$ W/cm². Fiber taper diameters from 20 µm to 125 µm are modeled by scaling up the PCF microstructure. We work in increments of 2.3 µm. The results for the average induced index change over the core region and the corresponding reflectivity are shown in Fig. 8. We split the graph of the reflectivity dependence on the fiber outer diameter into two segments (Fig. 8(b) and 8(c)) as the reflectivity abruptly increases in the range between 20 µm and 40 µm and we use different scales in order to reveal the details of the curves. In the simulations we used two fixed orientations of the PCF, i.e. along the ΓK and ΓM directions of the lattice.
Fig. 8. Dependence of (a) the average of the induced index change in the core region and (b), (c) the resulting grating reflectivity on the fiber taper diameter for gratings written in PCF-2 along the $\Gamma K$ and $\Gamma M$ directions of the hexagonal lattice.

For beam incidence along both the $\Gamma K$ and $\Gamma M$ directions there is a considerable increase of the induced index change and reflectivity for PCF taper diameters below 40 $\mu$m. At PCF taper diameters around 40 $\mu$m the diameter of the core region is a little less than 4 $\mu$m and very close to that of PCF-1 considered before. So both PCFs (PCF-1 and PCF-2) exhibit high refractive index change values when the core diameters are close to each other. Hence, we can assume that for the given waist radius of the writing beam (2.4 $\mu$m) efficient grating inscription takes place when the PCF core diameter is becoming comparable with the dimensions of the writing beam. This assumption needs further confirmation, which we will address in our future research.

For the $\Gamma M$ orientation the induced index change is mostly 0 due to poor transverse coupling. It increases for taper diameters around 55 $\mu$m and reaches its maximum value of $1.6 \times 10^{-4}$ for the smallest taper diameter considered here (22 $\mu$m).

We have also modeled the reflectivity of the resulting grating as done in Section 3. For most of the taper sizes, the reflectivity predicted by the average value of the induced change is higher than that calculated with the non-uniform index change. For the $\Gamma K$ orientation and for a taper size around 52 $\mu$m this picture changes. The reflectivity of the non-uniform grating is higher. A closer look to the profile of the index change revealed a high index region (above the Type II threshold) located in the central part of the PCF core, therefore yielding a large overlap with the fiber mode.

A similar phenomenon is observed for the $\Gamma M$ orientation around a taper diameter of 25 $\mu$m, where the reflectivity of the actual grating is higher than that predicted by the average value.

The importance of the location of the index change region and its overlap with the guided mode is particularly illustrative for a taper diameter around 21 $\mu$m, where the average of the induced index change for the $\Gamma K$ orientation is almost twice as large as that for the $\Gamma M$ orientation. In spite of that the reflectivity of the grating at the $\Gamma M$ orientation is slightly
higher than the one for the ΓK orientation, which again emphasizes the importance of index change region.

For the taper region from 125 μm to 40 μm depicted in Fig. 8(c) the highest reflectivity is observed at a taper diameter of 75 μm for the ΓK orientation, although the value is still only 4.8x10^{-3}. However this is an order of magnitude higher than the one of the non-tapered PCF.

![Fig. 9. Refractive index modification profile modeled for different values of fiber taper diameter for PCF (ESM-12-01) orientation along (a) ΓK (Media 1) and (b) ΓM direction of hexagon (Media 2).](image)

To better illustrate the index change profile in the PCF core region versus the taper diameter we refer to separate animations for both ΓK and ΓM orientations (see Fig. 9(a) (Media 1) and 9(b) (Media 2)). The maximum of the color scheme for the induced refractive index change in the movie is set to 10^{-3}. In the animations only the 6 holes of the first ring of hexagonal lattice microstructure are illustrated. In the animation we clearly see that for the ΓK orientation the index change takes places essentially at the edges of the core region resulting in weak coupling, while for the ΓM orientation more index change is observed in the central part, especially for small taper diameters. For some taper values we also see that those changes exceed the Type II threshold (changes in dark red color).

5. Conclusions

In this report we proposed, for the first time to our knowledge, a consistent methodology for modeling the refractive index modification profile and the resulting reflectivity of a grating inscribed in the core region of a PCF with femtosecond laser pulses at 800 nm and taking into account the non-linear nature of the refractive index change.

We applied our model to two PCFs and found that due to the intensity redistribution in the core region the average induced index change can be higher than for inscription in conventional fiber due to the highly non-linear nature of the 5-photon absorption process. We also showed that Type II index changes can be detected in the core region even though the peak intensity of the writing beam is well below this threshold.

We have also studied the influence of the angular orientation of the PCF with respect to the inscribing beam on the average induced index change. For the few-holed PCF with two rings of air holes we detected relatively high index changes for all orientations, although we observed a high dependence on the orientation as well. For commercially available ESM-12-01 PCF we observed no index change over a broad range of orientations (over 30°). Those results are in agreement with the experimental reports for this PCF, where the importance of accurate orientation was emphasized [21]. The differences in induced index change of almost an order of magnitude for two different PCF designs suggests that the efficiency of FBG inscription can be increased significantly by using PCFs with a design dedicated to grating writing.
Using coupled mode theory we have also investigated the influence of the non-uniform nature of the induced index change on the eventual grating reflectivity. For the given PCFs, we found that using the average values of the index change and neglecting its non-uniform nature leads to overestimating the reflectivity values. High index changes can be located at the edges of the core region where the small overlap with the guided mode will result in a much lower reflectivity. This also means that when estimating the index modulation of a FBG inscribed in a PCF based on the reflectivity of the grating as done for regular fibers, one should realize that the true value of the average index change is probably larger.

Finally we studied the influence of tapering on grating inscription for the commercially available ESM-12-01 PCF. For PCF taper diameters ranging from 125 μm to 40 μm no significant increase of the induced index change and reflectivity was observed, while in a range from 40 μm to 20 μm an order of magnitude increase in the average of induced index change was observed. This agrees well with earlier reports on PCF tapers with similar dimensions in which gratings were successfully fabricated [21,22]. The resulting reflectivity increased by two orders of magnitude, leaping from 0.005 at 40 μm to 0.3 at 20 μm. For most taper diameters the actual reflectivity was lower than that estimated by the average index change, which again emphasizes the necessity of taking into account the non-uniform nature of the index distribution. For inscription along the ΓM orientation lower induced index change values were observed compared to the ΓK orientation, although coupling with the guided mode was much better due to the centrally located index modification.

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