Estimation of Energy Minimizing Series Elastic Spring Stiffness for an Active Knee Prosthesis
Flynn, Louis; Geeroms, Joost; Heins, Sophie; Vanderborght, Bram; Lefeber, Dirk

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Abstract—Minimizing the mechanical motor output work or power required to create the desired actuator behavior in a quasi-static manner is a widely used, but incomplete method for actuator design. Here we use a dynamic electric and mechanical motor model combined with constrained output kinematics to find spring parameters for an active knee prosthesis using a series elastic actuator. This simulation was used to examine the landscape of motor torque tracking ability, motor electrical consumption, and the mechanical output of the system to determine spring stiffness ranges that should be further examined in the real device. Even though the task is negative net work, the actuator seems to benefit from having a plastic design, with a range of stiffness that improve energy consumption and torque tracking compared to a stiff device.

I. INTRODUCTION

The knee actuator that is the focus of this study was originally designed using a quasi-static approximation of the knee torque/angle relationship during overground normal walking, which determined the passive behavior and stiffness of the output springs [1]. Little attention was spent on the dynamics of the motor or output link to simplify the device analysis. It was assumed that when the output power and energy requirements of the motor were minimized by the series spring, a specific motor could be matched to the behavior of the output spring. This has, in the past, been a typical way of determining the series spring, as has been done in many studies such as [2], [3]. As a first approximation this can work, but it is not necessarily indicative of the electrical work of the motor. Other studies use a motor model and kinematically clamp the output to biological data such as in [4], [5], [6], [7], [8]. Here a dynamic model method with a simple controller is used to analyze the CYBERLEGs knee actuator to attempt to determine the best spring stiffness for walking behavior.

The motor model is especially needed when designing an actuator for a joint that is being used in a negative work situation. In a positive work generating joint, the motor must provide a holding torque so the spring can be deflected by forces on the output of the actuator, storing the energy in the series spring. A small additional displacement from the motor then adds the desired positive energy. In contrast to the ankle, the knee is not typically a power generating joint during normal walking. A typical average 60kg person dissipates energy on the order of 10 J/stride at the knee [9].

This study was designed to understand the behavior of the series elastic actuator, in particular to determine a flexion spring stiffness of the CYBERLEGs Gamma-Prosthesis knee under conditions that require the knee actuator to dissipate energy. In general, this method could be used to study any series elastic actuator that is intended to be used with a periodic output behavior, and some issues with this type of output kinematics constrained analysis are discussed. It should be noted that this is not a method to optimize the control parameters or control behavior but rather get an idea what the best physical parameters would be for energy minimization. Here we create a dynamic model of the knee prosthesis including the electrical and mechanical dynamic model of the knee motor and drivetrain with a kinematically fixed output to estimate the springs that create the most efficient gait in terms of electrical energy (Section II). Results of these simulations are then presented (Section III) followed by a discussion (Section IV) and conclusion (Section V).

II. METHODS

This study focuses on the dynamical model of the knee actuator through the use-case of a particular prosthesis design that was conceptually the same as the prostheses discussed in [1], [10]. Figure 4 shows a block diagram for the flow of the simulation. The analysis of the actuator is started by first determining the target output torque and kinematics for the periodic cycle to be optimized. In this simulation, the Winter average normal torque and kinematic trajectories for normal walking speed for an 60kg person [9] were used as targets.

The motor position is then precalculated to exactly track the torque output given the constrained kinematics of the output, as seen in the left side of Figure 4. The calculation of the desired actuator position trajectories was done using a quasi-static inverse torque model of the prosthesis found in [10], where the prosthesis geometrical characteristics are also found. This is not presented here for brevity.

The precalculated motor position vector is then fed into a Simulink model of the motor and driveline. The motor model is a simple DC electric motor with torque feedback disturbance where the output is the knee carriage position. The knee carriage position is then used in the forward model of the knee to calculate the joint torque given the dynamic tracking of the desired motor position. This model does not contain the output link dynamics because in this

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Sophie Heins is with the Institute of Mechanics, Materials, and Civil Engineering; Institute of Neuroscience; and Louvain Bionics; Universite catholique de Louvain, Louvain-la-Neuve, Belgium

Bram Vanderborght is with the Institute of Mechanics, Materials, and Civil Engineering; Institute of Neuroscience; and Louvain Bionics; Universite catholique de Louvain, Louvain-la-Neuve, Belgium

Dirk Lefeber is with the Institute of Mechanics, Materials, and Civil Engineering; Institute of Neuroscience; and Louvain Bionics; Universite catholique de Louvain, Louvain-la-Neuve, Belgium

1Louis Flynn, Joost Geeroms, Bram Vanderborght, and Dirk Lefeber are with the Robotics and MultiBody Mechanics Research Group, Vrije Universiteit Brussel and Flanders Make, Brussels, Belgium l.flynn@vub.be
2Sophie Heins is with the Institute of Mechanics, Materials, and Civil Engineering; Institute of Neuroscience; and Louvain Bionics; Universite catholique de Louvain, Louvain-la-Neuve, Belgium

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simulation the output link angle is constrained to follow a kinematic trajectory. Therefore the dynamics of this link do not directly have an effect on the actuator torque or motion as any dynamic effects are handled by external forcing of the system.

The simulation output is the motor electrical energy, the Mean Average error of the output torque tracking, and the output side mechanical work, measured for different spring stiffnesses and controller gains. These allowed a better understanding of the energetic landscape created by these variables and is discussed.

A. Prosthesis Operation

This prosthesis knee, which can be seen in Figure 1 is a parallel, two series elastic actuator system, with one actuator providing energy to the joint and the other a passive spring positioning system. The main motive actuator is located on the front of the knee and contains a motor that drives a carriage with two series springs to in turn drive the knee joint. These springs are called the Baseline (Bl) and Extension (Ex) springs, with the Bl spring providing flexion moments, and the Ex spring providing extension torques. The Bl spring is the main focus of the study, attempting to find an energy minimizing stiffness. The Weight Acceptance (WA) spring is on the back of the knee, unilaterally constrained for extension moments only, and is driven to the desired position under no load conditions.

B. Weight Acceptance

The Weight Acceptance mechanism (WA) is a device that inserts and removes a spring from the knee joint in the beginning of the stance phase, seen at the rear of the knee in Figure 1. The spring stiffness of the WA was chosen based on the very linear torque/angle relationship during the loading and unloading of the knee during early stance. In the simulation early stance phase knee flexion and extension was set to best approximate the knee extension moment from 1% to 22.5% of the gait cycle. This captured most of the knee flexion/extension moment during this phase and minimized the amount the knee actuator needs to do in this period.

Figure 2 shows the effect of the WA on the required actuator knee torque.

![Figure 2](image)

**Fig. 2.** The total target knee torque and knee torque due to WA action. The WA was active from the start of the gait cycle to 22.5%, approximating the majority of the torque during early stance phase. The total target torque is shown in Red, and the expected WA component is shown in Black. The difference of these is the target actuator torque which is shown in dashed Blue.

After subtracting the torque due to the the WA system, the desired torque trajectory of the knee actuator can be determined, which in turn is translated to a position trajectory of the knee actuator carraige, which is then tracked in the simulation. Note that in this model the action of the WA is idealized and is not included in the energy consumption calculation.

C. Motor Model

The equations of the standard DC motor with a fixed field and disturbance feedback are

\[ V(t) - Ri - L \frac{di}{dt} = e(t) \]  
(1)

\[ J \ddot{\theta} + b \dot{\theta} = T(t) - T_d(t) \]  
(2)

where \( V(t) \) is the motor input voltage, \( R \) is the winding resistance, \( L \) is the winding inductance, \( e(t) \) is the back EMF voltage, \( J \) is the output inertia of both the motor windings and the output gearbox, \( b \) is the system friction constant, and \( T(t) \) is the output torque. \( T_d(t) \) is the output disturbance, modeling the interaction force of the joint with the outside world.

Given that

\[ e(t) = K_e \frac{d\theta}{dt} \]  
(3)

\[ T = K_i \dot{i} \]  
(4)

and

\[ K_e = K_i = K \]  
(5)

and taking the Laplace transform, the system can be written as

\[ (Ls + R)\vec{T}(s) = \vec{V}(s) - KS\vec{\theta}(s) \]  
(6)

\[ (Js + b)s\vec{\theta}(s) = K\vec{T}(s) - T_d(s) \]  
(7)

Which in block diagram form can be written as in Figure 3.
D. Device Configuration

The motor used in the simulations included a maxon EC-4-pole 30, 200W, 36V motor (305014) in combination with a maxon Spindle Drive GP 32S, 1:1, 10X2mm spindle (363970). The values used in the simulation can be found in Table I. This motor/gearbox combination can reach both the velocity and torque required to track the Winter targets for normal walking within the physical device battery and current limitations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Units</th>
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<tr>
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<td>V/rad/sec or Nm/A</td>
</tr>
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<td>H</td>
</tr>
<tr>
<td>$b$</td>
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<td>$p$</td>
<td>0.002</td>
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<tr>
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**TABLE I**

PARAMETERS USED IN THE DYNAMIC SIMULATION OF THE KNEE ACTUATOR. THE FRICTION VALUE WAS TAKEN FROM SIMULATIONS WITH ACTUAL DATA MEASUREMENTS AND IS A REASONABLE ESTIMATE OF THE ACTUAL TOTAL SYSTEM FRICTION.

E. Motor Controller

The knee actuator motor is controlled by a simple proportional controller with gain of $P_{Gain}$. The output of this controller is the input voltage to the motor model, saturated at the maximum supply voltage ±36V. Because the simulation compared the same controller with different spring stiffness, it should be relatively indicative of the relative power consumption of the different configurations. The physical system has a few levels of additional complexity, including a low level current and velocity loop that are embedded in the motor driver. It should be clear that the power to run these additional physical controllers and motor drivers is not included in the simulation, as it only captures the actual motor power and there is no electrical model for the WA system. In reality it is likely that the controller dynamics will have an effect on the selected output spring stiffness, but this is beyond the scope of the paper.

**F. Simulation Conditions**

The simulation was run over two strides and the data for the second stride was used for analysis. The simulation was shown to reach steady state by the second step, avoiding transients from the first stride. Additional strides did not show significant differences from the second stride.

The simulation was run with different controller gains ($P_{Gain}$) ranging from 1 to 4 · 10^4, with a number of different Baseline spring stiffnesses ($K_{Bl}$) ranging from 1.5 · 10^4 to 2 · 10^5 N/m. The baseline stiffness were chosen based on the minimum stiffness required to create the torque trajectory given the physical actuator’s range of motion and a high enough stiffness to understand the general behavior. All combinations were tried in a brute force search method to examine the shape of the output landscape.

III. RESULTS

Here the ability of the actuator motor to track the desired motion is examined, which can be determined by comparing the output torque to the desired torque. This can then be compared to the electrical energy required to move the actuator to track the trajectory as well as the actual output mechanical energy, which is the energy needed to force the output to track the desired kinematics. By comparing these three characteristics, a suggested Baseline spring constant and controller gain can be determined.

A. Actuator Torque Mean Absolute Error

First we can examine the Mean Absolute Error (MAE) to gauge how well the system tracks the desired torque trajectory over the range of selected controller gains and $K_{Bl}$. As a measure of the torque error of the controlled joint, the Mean Absolute Error was used, as in Equation 8.

$$MAE = \frac{1}{n} \sum_{j=1}^{n} |\tau_{des} - \tau_{act}|$$

This was used opposed to Root Mean Square Error (RMSE) due to the fact that RMSE places a high weight on large errors, even if they are for a short duration. In this application, it was felt that having a short duration of large error was less detrimental than having a larger average error during extended periods of the gait cycle.

Figure 5 shows the landscape of the MAE at a number of different controller gains and Baseline spring stiffness. Note that the absolute minimum value for MAE for this simulation comes at a $P_{Gain}$ of 4 · 10^4 and a $K_{Bl}$ spring stiffness of 1.1 · 10^5 N/m. The dashed red line in the Figure shows the minimum at each of the controller gains. This trend continues if the gains are extended past the range of the simulation, following a spring constant of about 1.1 · 10^5 N/m until the proportional controller becomes unstable at a $P_{Gain}$ of about 7 · 10^4. The solid red line in Figure 5 shows the spring constant that was used in the original device (2.14 · 10^4 N/m) which was determined from the passive Winter data including a mechanism for removing energy from the joint.
But to say the best behavior happens at the absolute minimum gain/spring constant combination is a bit misleading, as not all controller gains are stable to step response inputs with an unconstrained output. Because the simulation waits for the second step to reach steady state, the output is kinematically constrained, and the desired output kinematics are relatively smooth and have gradual changes, the gain can be pushed much higher than what is realistic for stable response to an unconstrained step input. The $P_{Gain}$ that leads to a reasonable overshoot and settling time lies in the region between $4 \cdot 10^3$ to $7 \cdot 10^3$, depending on the criteria for overshoot. In the Figures 5, 7 and 8 this gain limit is marked by a green line.

If the spring for the fastest response and minimum error without going unstable is chosen, the result would be $K_{Bl} = 5.5 \cdot 10^3 \text{N/m}$, $MAE = 2.1566 \text{Nm/stride}$. This would result in a spring constant about two times as stiff as the original springs.

Note that the choice in extension spring stiffness does not make a large change in the behavior of the device in terms of MAE. Figure 6 shows a close up of the region of interest, of gains below $7 \cdot 10^3$ and $K_{Bl}$ less than $6 \cdot 10^3 \text{N/m}$. The minimum MAE is once again marked showing a similar behavior as before.

Figure 7 shows the landscape from the motor electrical energy point of view. Here the total motor electrical energy, which is measured by integrating the product of the winding voltage and current over a stride, is shown with respect to the same $K_{Bl}$ and $P_{Gain}$ ranges as previously. Here the minimum energy value (yellow dot) in the Figure resides in a highly negative region ($-17.6 \text{J/stride}$); the knee is actually harvesting energy because the output forces the knee backwards. In fact the output mechanical energy of the device at this point, measured by the actual output torque and joint displacement, is $-59 \text{J/stride}$, as seen in Figure 8. This happens because the knee isn’t following the Winter targets in this region as noted in the rise in the MAE (see figure 5), and because the output is kinematically fixed rather than, for example, using a torque constraint, the output can drive a considerable amount of power back into the knee.

As the MAE decreases the output power tends to the Winter average, as can be seen at the minimum MAE point.
region is defined by a spring stiffness around $5.5 \cdot 10^4 N/m$, which is twice as stiff as the previous springs that were used, and a range of gains up to $P_{Gain} = 7 \cdot 10^3$.

The knee is a dissipative joint requiring the removal of about $10 J/stride$, according to average gait data for a $60 kg$ person. This is over half of the energy needed by the ankle during the gait cycle, and original designs of the CYBERLEGs knee actuator attempted to capture this negative work of the knee and distribute it directly to the ankle through a device called the Energy Transfer mechanism (ET). This proved to be difficult in practice to do reliably, and so the device was removed requiring the knee actuator to dissipate the entirety of the knee work. Without an ET, it is not as clear how the series elastic actuator would benefit the knee if acting without the energy dissipation the ET provided.

The simulation shows the device becomes unstable at low stiffness and both low and high gains. At very low stiffness ($< 1.5 \cdot 10^4 N/m$) the springs simply are not capable of providing the required torque and are disregarded. At low stiffness, the actuator is not able to provide the correct velocity and energy is wasted trying to track the correct actuator position, ultimately resulting in full current demand nearly the entire time and poor output tracking. In high stiffness regions, small errors in the actuator position result in larger torque errors. In low gain regions, the system cannot correctly track, and due to the fact the output is kinematically constrained, the output drives the joint backwards. In high gain regions, the system becomes unstable and begins to oscillate.

**A. Model Control**

The controller for this model has intentionally been kept simple. First, this model is based on an open-loop position control of the actuator, and not on a torque or output position feedback control loop. For systems looking to provide specific output torques there are many methods to examine ([11], [12]), many of which use cascading control loops that are much closer to the actual controller used in the hardware. To achieve better performance in this simulation, one could optimize a PID or model based controller for every baseline spring configuration, which will likely improve the tracking accuracy across the entire range. This wasn’t done due to the added complexity required to correctly tune a complicated controller at each stiffness, where the tuning parameters could have a dramatic effect on the energy consumption if not done carefully but could be better addressed in future simulations. Here we assume that the general behavior of the overall landscape will remain relatively similar, but realize that the real system will behave differently depending on the dynamics of the actual controller. This simulation could eventually incorporate a real optimization of all of the parameters, including a check on the unconstrained system response, to find a better controller result.

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**IV. DISCUSSION**

Here we have presented a simulation of the knee actuator with weight acceptance mechanism controlled by a proportional controller on the actuator position, and show the regions of baseline spring stiffness that may be more efficient than the ones that were originally used and found using a quasi-static approach. Using the model, a region of spring stiffness and control gains has been determined that would be worth investigating for better actuator torque tracking. This (magenta, $-11.14 J/stride$). Note that the average knee work for a $60 Kg$ individual is about $-10 J/stride$. If we choose the spring for the fastest response and minimum error without going unstable, the result would be $-5.2 J/stride$, showing that ideally the knee would still be generating energy while producing the output torque using this method of simulation.

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**Fig. 7.** The electrical energy per stride measured at the motor as the integral of voltage and current. The yellow point is the point of Minimum MAE and the magenta point is the Minimum output energy configuration of the simulated region. The dashed red line is the minimum MAE at each gain. The red line marks the spring constant $(2.14 \cdot 10^4 N/m)$ of the knee as used, determined by a quasi-stiffness analysis of the knee behavior. The green line marks the gain limit $P_{Gain} = 7 \cdot 10^3$.

**Fig. 8.** The mechanical work at the output of the Knee measured with actual output torque and joint displacement. The yellow point is the point of Minimum MAE and the magenta point is the Minimum output energy configuration of the simulated region. The dashed red line is the minimum MAE at each gain. The red line marks the spring constant $(2.14 \cdot 10^4 N/m)$ of the knee as used, determined by a quasi-stiffness analysis of the knee behavior. The green line marks the gain limit $P_{Gain} = 7 \cdot 10^3$. 


B. Assumptions of the Model

Using an open-loop actuator position control method assumes that the Winter torque targets are capable of producing the correct kinematics and joint torques when the person using the device is actually walking. This means that the output mass and inertia of the actuated link must be similar to a normal human leg, particularly during the swing phase. The DC motor model is relatively simple, and does not include many effects such as field coupling efficiency and temperature effects, but should contain many of the first-order properties. Also the choice to not change the extension spring constant was based on the fact that the WA takes care of the highest extension torque periods, and the others are not as large. Running the simulation with an extension spring that is equal to the stiffness of the baseline spring showed little difference in results.

C. Output Constraints

The choice of constraining the output kinematically has a number of effects on the knee behavior which do not represent the actual behavior of the device if the output is unconstrained. First, because the knee output is kinematically constrained, the output impedance is essentially infinite. This actually has some implications for the gain tuning, as the high output impedance reduces the overshoot because the output pushes the actuator back into position, rather than changing the output trajectory. Resonances that can occur from an oversensitive actuator are not seen in this method of simulation and results in a controller gain that looks stable, but in fact would not be with an unconstrained output. If the output is kinematically constrained, the system becomes unstable around $P_{Gain} = 6 \times 10^4$. Unconstrained, in this model, the proportional gains cannot exceed a $P_{Gain}$ of $7 \times 10^3$.

Another aspect of the actuator that is not explored in this simulation is the torque response to output disturbance. Because the output is kinematically constrained, there is no undesired output acceleration that would be required to be handled by the motor. It is well known that the stiffer the series spring the increase in torque error will be when disturbed from the output [13].

D. Comparison to Original Springs

The original spring stiffness chosen with the ET in place was considerably lower, at $2.14 \times 10^3 \text{N/m}$, and is marked by a red line in Figures 5, 7, and 8. In the actual actuator, this produces the knee quasi-stiffness during the knee extension at the end of swing phase when the motor is held in place. Looking at this region in the Figures, the MAE and the energy consumption are not as good as could be found with higher stiffnesses. Higher controller gains were extremely energetically costly, as the higher gains tended to saturate the actuator motor current trying to accelerate the motor and driveline inertia. This was very much how the actuator behaved in the real device, where high controller gains tended to be energetically costly, without great improvement in the tracking capability.

V. CONCLUSIONS

Examining the CYBERLEGs knee actuator with a full dynamic motor model and kinematically constrained output can allow the better choice of spring stiffnesses for better output torque tracking and energy consumption than simply minimizing the required motor mechanical output. Here it was determined that a stiffer Baseline spring than was originally selected would likely improve the behavior of the device, as well as reduce the actuator work, to produce the Winter average torque output even though the desired task requires a net dissipation of energy. There does seem to be an optimal region for the spring stiffness, as raising the stiffness above $5.5 \times 10^3 \text{N/m}$ slightly decreases the tracking accuracy of the device, and lowering it dramatically increases the energetic cost as well as reduces accuracy.

REFERENCES