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Published in:
Digital Holography & 3-D Imaging

Publication date:
2015

Document Version:
Accepted author manuscript

Link to publication

Citation for published version (APA):
Reconstruction Resilience to Subsampling in Compressive Fresnel Holography

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Abstract: The reconstruction resilience of a Projected Gradient Method (PGM), POCS and TwIST to randomly subsampled image wavefields is investigated. POCS and PGM do not consider noisy data, but they return a better reconstruction than TwIST.

OCIS codes: 090.1995 (Digital Holography), 100.3190 (Inverse Problems), 100.7410 (Wavelets).

1. Introduction

The propagation of coherent and monochromatic light in free space is modelled by a linear operator which maps an object wavefield $o$ to an image wavefield $i$ [1]. The latter is derived from one or more digitally recorded interferograms, which implies that $i$ is uniformly sampled in $N$ points. Lexicographical ordering of these samples results in a 1D discrete signal or vector $i$ with $N$ elements. Now, $o$ can be reconstructed with the inverse light propagation operator in $N$ points as well, which gives rise to the vector $o$. In the Compressive Sensing (CS) framework, $o$ is represented in a sparsifying basis $W$ and then reconstructed from less than $N$ measurements in the image plane [2]. A measurement $m$ is a linear combination of the elements in $i$ and $M$ measurements are stacked in a vector $m$ (with $M < N$ measurements). The linear relationship between $m$ and the sparsifying coefficients $x = Wo$ then yields:

$$m = Si = SFo = SFW^{-1}x,$$

where, in this work, $S$ is a subsampling matrix, $F$ models the propagation of light in free space and is implemented with the angular spectrum method, $W$ is a sparsifying basis of $o$ and $x$ is the collection of coefficients that identify $o$ in $W$. In prior work, $x$ has already been reconstructed from $m$ through the minimization of the objective function

$$f(x) = \frac{1}{2} \|Ax - m\|^2 + \tau \|x\|_1,$$

where $A = SFW^{-1}$ and $\tau$ is a positive regularization parameter. To solve this problem, Two-step Iterative Shrinkage/Thresholding (TwIST) with $\ell_1$-norm [3] and total variation [4] minimization was proposed. In this work, the Gini index is proposed as a sparsifying measure for justifying the choice of the orthogonal wavelet basis, a Projected Gradient Method (PGM) is introduced for solving (1) with regard to $x$, and the resilience of the reconstruction to random subsampling of the image wavefield is investigated for TwIST, PGM and Projection Over Convex Sets (POCS). For the details on POCS and TwIST we refer respectively to [5] and [6].

2. Methods

2.1. Estimation of the reference object wavefield

The ground truth object wavefield is only known in a simulation. In practice, one or more noisy holograms are recorded and these provide the necessary information to derive $i$, which is noisy as well. The subsampling matrix $S$ is the unit matrix when all samples in the image wavefield are considered. In that case, $SF$ is invertible and an estimate of the object wavefield is obtained from (1): $\hat{o} = F^{-1}i$. Throughout this work, $\hat{o}$ is the reference object wavefield.

2.2. The Gini index

The object wavefield is ideally decomposed in its most sparsifying basis. The Gini index [7] was selected in order to assess the degree of sparsity of $\hat{o}$ in each of the orthonormal wavelet bases in the Daubechies, Symlet and Coiflet wavelet families. The choice for this sparsifying measure is justified in [8].
2.3. Reconstruction algorithms

The noise can be neglected when the signal to noise ratio of the digital holograms is high. It then suffices to solve the optimization problem

\[ \arg\min_x \{ \|x\|_1 \} \quad \text{subject to} \quad Ax = m, \]  

with a PGM [9]. This is an iterative algorithm in which the solution after the \( k \)th iteration step is \( x_k \). The operator \( H \) takes the complex conjugate transpose and the generalized right inverse \( A^+ = A^H(AA^H)^{-1} \). The initial estimate \( x_0 \) is assigned \( A^+ M \). And in each iteration step, \( x_k \) is first shifted in the opposite direction of the subgradient of \( \|x\|_1 \):

\[ \tilde{x}_k = x_k - \alpha \nabla_{\text{sub}}(\|x\|_1) \big|_{x=x_k} = x_k - \alpha \text{sgn}(x_k), \]  

with \( \alpha \) a parameter and \( \text{sgn}(x) \) the sign function, followed by a projection:

\[ x_{k+1} = x_k - A^+(A\tilde{x}_k - m). \]  

The initial value of \( \alpha \) is one tenth of the mean of the absolute values of the elements of \( x_0 \), and \( \alpha \) is divided by 1.5 if \( \|x_{k+1}\|_1 \geq \|x_k\|_1 \). \( x_{k+1} \) is discarded in the latter case and recomputed for the new \( \alpha \). This way, a monotonically decreasing sequence of \( \ell_1 \)-norms is obtained and the algorithm stops when the relative error between the new solution and the mean of the last 3 solutions is less than 10\(^{-4} \). This stop condition was also adopted by POCS (similar to PGM, but the subgradient in (4) is replaced by a soft thresholding step) and TwIST. The latter, however, requires the selection of an additional parameter \( \tau \), which influences the solution. \( \tau \) was set to 0.005.

3. Experiments

The 980 x 1280 holograms were acquired in an off-axis geometry. The reconstructed image wavefields were mirror extended at the boundaries and cut to 1024 x 1024 pixels (\( N = 1024^2 \)). The estimates of the object wavefield \( \hat{o} \) of a USAF 1951 test target and a MEMS lens are shown in Figure 1 (a) in the regions of interest. The object wavefields were represented in a 4 level Mallat decomposition using the Daubechies 27 wavelets for the USAF test target and the Daubechies 29 wavelets for the MEMS lens. The maximum Gini indices were observed for these wavelets and yield \( GI_{\text{USAF}}(\text{db27}) = 0.61 \) and \( GI_{\text{lens}}(\text{db29}) = 0.85 \).

![Reference \( \hat{o} \) and reconstructed object wavefields from 10\% of the samples in the image wavefield](image)

Fig. 1: Reference \( \hat{o} \) (column (a)) and reconstructed object wavefields (columns (b), (c) and (d)) from 10% of the samples in the image wavefield for a MEMS lens (top row) and a USAF 1951 test target (bottom row).

The object wavefield was reconstructed with PGM, POCS and TwIST from a set of measurements that were randomly and uniformly selected from the samples in the image wavefield, \( |m| = 0.1 \ast 1024^2 \) and the subsampling ratio yields 0.1. Figure 1 shows that the reconstruction by means of TwIST yields the darkest background, which indicates that the energy is not fully restored. \( \tau \) was manually tuned in order to maximize the Peak Signal to Noise Ratio (PSNR)
between the reconstructed amplitude and the amplitude of \( \hat{o} \). This is a cumbersome procedure while PGM and POCS run autonomously and reconstruct more of the energy, despite neglecting the noise in (3). Furthermore, the details are slightly better reconstructed with POCS than with PGM. This is most obvious in the top left corner of the USAF test target. Next, the reconstruction with the three methods was repeated 10 times with independent realisations of the uniformly random subsampling matrix \( S \), for a subsampling ratio that goes from 0.1 to 0.9 in steps of 0.1. The mean PSNR between the reference and reconstructed amplitude object fields are shown in Figure 2.

![Fig. 2: Mean amplitude PSNR as a function of the subsampling ratio for the reconstruction of the USAF test target (left) and MEMS lens (right) with PGM (●), POCS (▲) and TwIST (♦).](image)

In Figure 2, it is confirmed that the reconstruction with TwIST yields the worst mean PSNR for all subsampling ratios despite the tuning of \( \tau \). The mean PSNR is systematically higher for PGM and POCS, which are in general well-matched. Hence, neglecting the noise on the image wavefield was beneficial for the reconstruction of the object wavefield in comparison with solving (2) and tuning \( \tau \). Finally, the standard deviation in all sets of 10 experiments is maximum 0.16 dB, which validates the repeatability of the experiments for different subsampling realisations.

4. Conclusion

We have demonstrated that the reconstruction of the object wavefield from a subsampled image wavefield is faster and better in terms of the PSNR for POCS and PGM than for TwIST with \( \ell_1 \)-norm minimization. This justifies that the noise could be neglected.

5. Acknowledgements

The research leading to these results has received funding from the European Research Council under the European Union’s Seventh Framework Programme (FP7/2007-2013)/ERC Grant Agreement n. 617779 (INTERFERE), the Agency for Innovation by Science and Technology in Flanders (IWT) and the National Natural Science Foundation of China (Grant No. 61405111).

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