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Noise leakage suppression in VCO-based $\Sigma\Delta$-modulators excited by modulated signals

Dries Peumans, Piet Bronders and Gerd Vandersteen

***Abstract*—Oversampling data converters, such as the $\Sigma\Delta$-modulator, are widely used for analog-to-digital conversion. Proper evaluation of the in-band noise densities and the corresponding effective number of bits requires a post-processing step to adequately suppress the noise leakage. This suppression is traditionally achieved by windowing the digital output signal. Although windowing is effective for sinusoidal excitations, it is less optimal for the analysis of modulated signals. We propose a technique that not only accurately derives the in-band noise densities for these modulated signals, but also provides a measure for the in-band nonlinear distortions generated by the modulator. The performance of the technique is demonstrated on a $\Sigma\Delta$-modulator employing a voltage controlled oscillator for the internal quantiser.**

***Index Terms*—$\Sigma\Delta$-modulator, noise leakage suppression, local polynomial method**

I. Introduction

The rapid downscaling of CMOS processes has resulted in an increasing demand for highly digital Analog-to-Digital Converters (ADCs). Incorporation of a ring-based Voltage-Controlled Oscillator (VCO) in the quantiser of a continuous-time $\Sigma\Delta$-ADC alleviates the problems encountered with limited voltage headroom and effectively narrows the border between the analog and digital world [1]–[3].

Evaluation of the performance of these VCO-based $\Sigma\Delta$-ADCs is traditionally accomplished by application of elementary signals such as single-tone or two-tone excitations [4]. Unfortunately, modern telecommunication signals (e.g. OFDM) are becoming more and more sophisticated and require that we use these modulated signals during simulation such that a correct representation of the actual performance can be obtained. At first sight, using these modulated signals does not necessitate the use of advanced post-processing techniques to derive the output spectrum and in-band noise densities. However, applying the traditional windowing techniques [4] on the digital output signal imposes some limitations:

- Depending on the window used, a certain minimal amount of periods (> 2) of the modulated signal should be acquired to derive the in-band noise densities.
- No information about the in-band non-linear distortions can be extracted.

In recent years, local modelling techniques such as the Local Polynomial Method (LPM) have been introduced that cope with these limitations [5]. The LPM starts from measured (or simulated) noisy input-output data and locally approximates the transfer function and noise leakage term by a polynomial. By doing so, the leakage suppression is improved compared to windowing. Additionally, the level of in-band nonlinear distortions can be derived [6]. Thus far the LPM has only been applied onto systems which completely reside in the analog or digital domain. In this paper, we apply the LPM for the first time on an inherent mixed-signal system, i.e. the VCO-based $\Sigma\Delta$-modulator, and showcase the benefits of this method compared to windowing.

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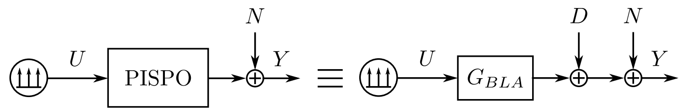

Fig. 1: A time-invariant PISPO system can be approximated by the Best Linear Approximation G_{BLA} and an additive nonlinear distortion source D.

II. The Local Polynomial Method

The purpose of all the local modelling techniques is to retrieve the so-called Best Linear Approximation (BLA) G_{BLA} of a time-invariant period-in same period-out (PISPO) system (Fig. 1). This PISPO class of systems encompasses a wide class of dynamic nonlinear systems but rules out chaotic systems or systems which generate sub-harmonics [6]. It is assumed that the system is excited by a modulated signal with a fixed Power Spectral Density (PSD) and a fixed Probability Density Function (PDF). In this paper, we use random-phase multisines to emulate the behaviour of these modulated signals. These multisines are preferred due their complete control over the PSD, while their periodic nature grants nonlinear detection capabilities [6]. They are defined in the time-domain as the sum of F harmonically related sines

$$u(t) = \sum_{k=1}^{F} A_k \, \sin(2\pi k f_{res}\, t + \varphi_k) \qquad (1)$$

where A_k and φ_k are, respectively, the amplitude and phase of the k^{th} component and f_{res} is the frequency resolution of the multisine. A_k determines the PSD of the multisine and can be tailored to resemble the actual modulated signals present in the application at hand. When φ_k is chosen uniformly in $[0, 2\pi[$, a random-phase multisine with a Gaussian PDF is obtained [7]. Signals with a Gaussian PDF are especially important in telecommunications since they maximise the entropy and consequently the transmitted information content.

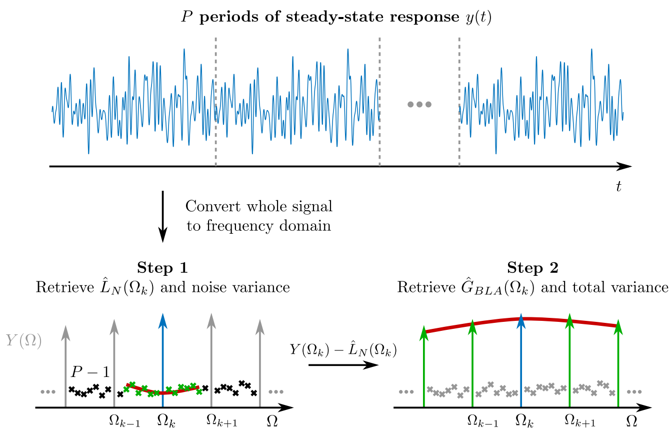

Fig. 2: The LPM makes use of a two-step procedure to remove the noise leakage and at the same time retrieve an estimate for the in-band noise variance and in-band nonlinear distortion variance. ↑: excited frequency for which the estimation is performed, ↑ or ×: points taken into account for the local modelling, −: estimated polynomial model. The local modelling strategy is repeated for every Ω_k.

The BLA of the system approximates best, in least-squares sense, the linear behaviour of the PISPO system and collects all the nonlinear contributions introduced by the system in a single additive nonlinear distortion source D. In the frequency domain, the input-output behaviour of the PISPO system can be exactly described by the following fundamental equation in function of the generalised frequency variable Ω

$$Y(\Omega) = G_{BLA}(\Omega)\,U(\Omega) + D(\Omega) + N(\Omega) + L_N(\Omega) \quad (2)$$

where $U(\Omega)$, $Y(\Omega)$ are respectively the input and output of the system in the frequency domain and $N(\Omega)$ is an additive noise source which in general represents the measurement noise or, in the case of the $\Sigma\Delta$-modulator, corresponds with the quantisation noise introduced by the quantiser. Due to the non-periodic nature of the noise, a noise leakage term $L_N(\Omega)$ is added as an unwanted contribution to the output. Depending on the domain of the signals, Ω equals $j\omega$ for continuous-time systems and $e^{-j\omega T_s}$ for discrete-time systems where T_s represents the sampling period and ω is the angular frequency. The extension towards mixed-signal systems is explained in Section III.

The key idea behind the LPM is to approximate both $G_{BLA}(\Omega)$ and $L_N(\Omega)$, which are presumably smooth functions [6], with a local polynomial model in Ω at each excited frequency Ω_k (Ω_k is evaluated at $\omega_k = 2\pi k f_{res}$)

$$G_{BLA}(\Omega) = \hat{G}_{BLA}(\Omega_k) + \sum_{i=1}^{n_G} g_{ki}\,(\Omega - \Omega_k)^i \quad (3)$$

$$L_N(\Omega) = \hat{L}_N(\Omega_k) + \sum_{i=1}^{n_L} l_{ki}\,(\Omega - \Omega_k)^i \quad (4)$$

where $\hat{G}_{BLA}(\Omega_k)$ and $\hat{L}_N(\Omega_k)$ are, respectively, the local BLA and noise leakage estimates while g_{ki} and l_{ki} are the unknown polynomial coefficients which model the dynamic variations within each local model. Since these variations are limited in the local modelling context, the polynomial orders n_G and n_L can be chosen small ($<= 4$). Substituting (3) and (4) in (2) results in a linear set of equations which can be solved with a linear least squares estimation procedure [8]. Unfortunately, application of the procedure in this manner would not allow us to make a distinction between the noise $N(\Omega)$ and the nonlinear distortions $D(\Omega)$ since both behave unpredictably in funcion of Ω. Therefore, a two-step procedure has been proposed [5] which makes explicit use of the periodicity of the random-phase multisine and has the ability to distinguish the noise from the nonlinear contributions (Fig. 2).

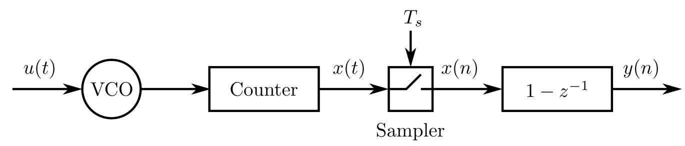

Fig. 3: General schematic of the VCO-based $\Sigma\Delta$-modulator with first order noise shaping.

Step 1*: Retrieve $\hat{L}_N(\Omega_k)$ and noise variance*

The procedure starts from $P >= 2$ periods of the input-output steady-state response to a random-phase multisine (top of Fig. 2). Converting the assemble of these P periods to the frequency domain results in a spectrum where $P-1$ additional points appear between the original excited frequency lines Ω_k. Contributions of the BLA $G_{BLA}(\Omega)\,U(\Omega)$ and the nonlinear distortion $D(\Omega)$ only appear on the excited frequencies Ω_k in (2), implying that these additional points only contain contributions of the noise $N(\Omega)$ and noise leakage $L_N(\Omega)$. Using (4) in a local band around Ω_k consequently allows us to retrieve $\hat{L}_N(\Omega_k)$ and the sample noise variance using linear least squares:

$$\mathrm{Var}\{N(\Omega_k)\} = \frac{1}{q} \sum_{\Omega \in S} |Y(\Omega) - L_N(\Omega)|^2 \quad (5)$$

where S is the set of neighbouring frequencies used during the local model estimation (× in Fig. 2) and q is the number of degrees of freedom [6].

Step 2*: Retrieve $\hat{G}(\Omega_k)$ and total variance*

Using the results from step 1, the estimated noise leakage $\hat{L}_N(\Omega_k)$ can be subtracted from $Y(\Omega_k)$ and results in the corrected spectrum $Y_{corr}(\Omega_k)$. By substituting (3) in (2), a linear set of equations is obtained which uses this corrected spectrum at the surrounding excited frequencies to derive an estimate for G_{BLA} and the total variance $\mathrm{Var}\{D(\Omega_k)\} + \mathrm{Var}\{N(\Omega_k)\}$:

$$\frac{1}{q} \sum_{\Omega \in R} |Y_{corr}(\Omega) - G_{BLA}(\Omega)\,U(\Omega)|^2 \quad (6)$$

where R is the set of excited frequencies used during the local estimation (↑ in Fig. 2). The variance of the nonlinear distortions can be calculated by subtracting $\mathrm{Var}\{N(\Omega_k)\}$ from the obtained total variance.

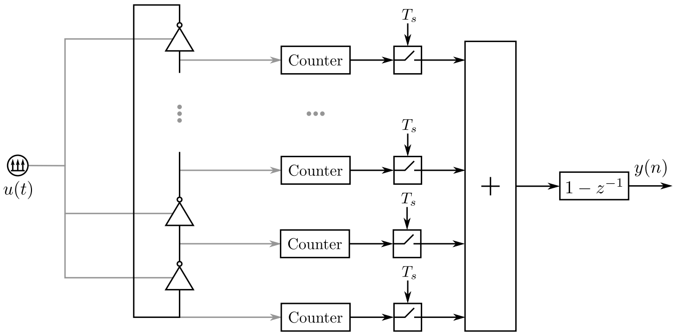

Fig. 4: Multiphase implementation of the VCO-based $\Sigma\Delta$-modulator increases the resolution of the ADC by sampling every intermediate stage of the voltage-controlled ring oscillator independently.

III. Applying the LPM on mixed-signal systems

One of the major assumptions of the LPM is that the input and output signals reside in the same domain. However, a VCO-based $\Sigma\Delta$-modulator has a continuous input and a discrete output, making it a mixed-signal system (Fig. 3). To decide whether or not the LPM can still be applied to this type of system, we need to take a look at the equation in the frequency domain which relates the analog signal prior to the sampler $x(t)$ and the discrete output $y(n)$ [9]

$$\mathcal{Z}\{y(n)\} = (1 - z^{-1})\ \mathcal{L}\{x(t)\,\delta_{T_s}(t)\}(s)\Big|_{z=e^{sT_s}} \tag{7}$$

where $\mathcal{Z}\{\bullet\}(z)$ is the Z-transform with variable z, $\mathcal{L}\{\bullet\}(s)$ is the Laplace transform with variable s and $\delta_{T_s}(t)$ is a Dirac comb with period T_s. From (7) it can be observed that a transformation $z = e^{sT_s}$ is needed to convert from the analog to the digital domain. In the most general case, this transformation does not allow us to merely use polynomials in one variable, i.e. s, to describe the mixed-signal behaviour. Luckily, the dynamic variations encountered by the LPM are limited due to the narrow local window and consequently it makes sense to expand $z = e^{sT_s}$ into its polynomial series expansion around each excited frequency Ω_k

$$z = e^{sT_s} = e^{j\Omega_k T_s} \sum_{n=0}^{+\infty} \frac{(s - j\Omega_k)^n\, T_s^n}{n!} \tag{8}$$

Following this reasoning, the LPM can therefore be applied as is since the mixed-signal system can be locally approximated by polynomials in s.

IV. Example: VCO-based $\Sigma\Delta$-modulator

The described method is applied to a first-order VCO-based $\Sigma\Delta$-modulator and the results are compared with the windowing technique (Hanning window). The $\Sigma\Delta$-modulator is designed in Agilent's Advanced Design System (ADS) using a $0.18\,\mu$m CMOS process and consists of: (i) a current-starved voltage-controlled ring oscillator with 7 inverter stages, (ii) a counter and sampler which discretise in time every intermediate stage, (iii) a summator, and (iv) a discrete-time filter for first-order noise shaping [1]. The sampling frequency of the modulator is set to $1\,$GHz. To avoid aliasing errors, care has been taken during the design of the VCO such that the oscillation frequency does not exceed $500\,$MHz. The multisine applied to the system has a root mean square value of $0.1\,$V and covers a bandwidth of $10\,$MHz with a frequency resolution f_{res} equal to $250\,$kHz. A transient simulation is used to retrieve four periods of the steady-state response to the multisine excitation. The obtained data was post-processed in Matlab.

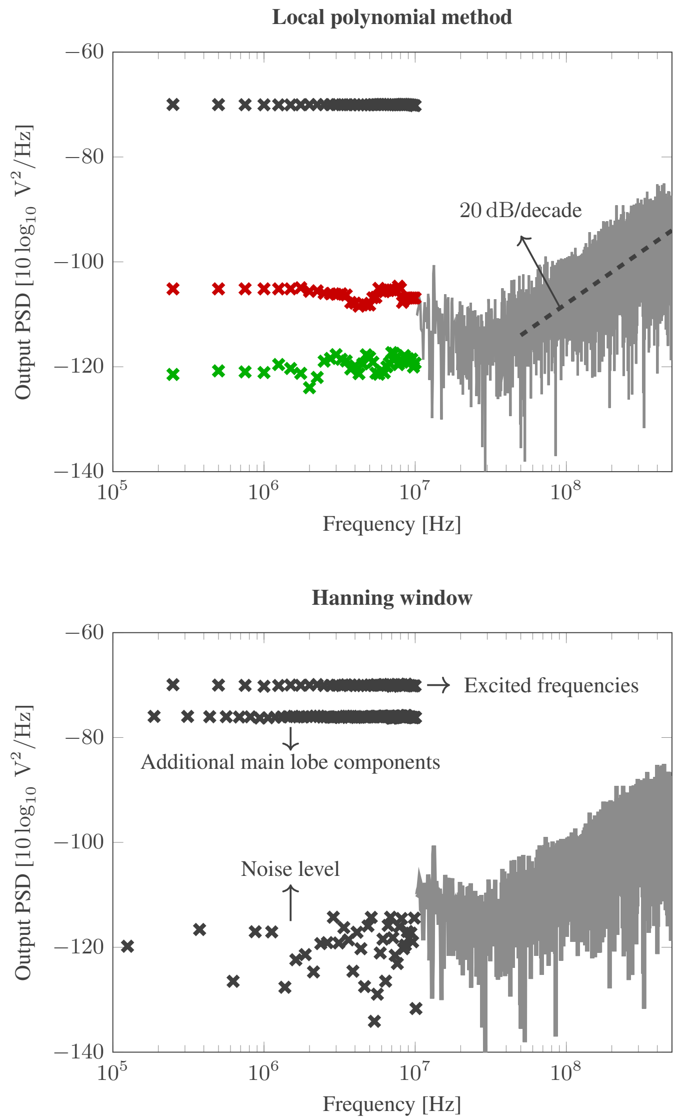

Fig. 5: Comparison between the output PSD obtained after application of the LPM and the Hanning window (×) using 4 periods of the steady-state response. Additionally, the estimated in-band total variance (×), which includes the nonlinear distortion, and in-band noise variance (×) retrieved with the LPM are shown. The out-of-band quantization noise (—) is subject to first-order filtering.

Application of the LPM ($n_G = n_L = 2$) and the Hanning window on the retrieved data reveals the differences between both methods (Fig. 5). The LPM provides an accurate estimate of the output PSD, the in-band noise variance and in-band total

Periods P	LPM	Hanning	Hamming	Blackman
2	-119.76	-	-	-
3	-120.19	-	-	-
4	-120.68	-122.25	-122.85	-
5	-121.93	-122.52	-123.13	-
6	-122.32	-123.38	-123.99	-122.44
7	-122.59	-123.96	-124.49	-123.71

Table I: Comparison between the average in-band noise PSD ($10\log_{10}$ V^2/Hz) obtained with the LPM and three commonly applied windowing techniques (Hanning, Hamming and Blackman) for different number of periods P.

variance starting from a minimum of two periods. Differences between these two variances are due to the nonlinear behaviour of the modulator. The Hanning window generates unwanted contributions right next to the excited frequencies due to the extended width of the main lobe of this window's frequency response. Therefore, at least four periods are needed to reveal the in-band noise with this window. Additionally, remark that the Hanning window only provides information about the in-band noise contributions and completely neglects the nonlinear distortions produced by the system.

To further evaluate the performance of the LPM compared with commonly applied windowing techniques (Hanning, Hamming and Blackman) [10], we determined the average in-band noise PSD for different number of periods of the steady-state response (Table I). From Table I, it can be concluded that the estimates for the average in-band noise PSD do not differ significantly (maximum difference for $P = 4$ is $2.17\,\mathrm{dB}$). However, depending on the width of the main lobe of the window used, the minimal amount of periods to acquire the in-band noise PSD changes and potentially negatively affects the simulation or measurement time. For example, the Blackman window requires a three times larger simulation time to assess the in-band noise PSD.

V. Conclusion

In this work, an accurate technique for noise leakage reduction in VCO-based $\Sigma\Delta$-modulators excited by modulated signals has been introduced. Using multisine excitations, the proposed technique allows us to lower the simulation time compared to the widely used windowing techniques. Additionally, estimates for both the in-band quantisation noise and in-band nonlinear distortions can be calculated. Due to these attractive features, the proposed technique contains all the properties to be a valid alternative to windowing techniques.

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References

[1] M. Z. Straayer and M. H. Perrott, "A 12-Bit, 10-MHz Bandwidth, Continuous-Time $\Sigma\Delta$ ADC With a 5-Bit, 950-MS/s VCO-Based Quantizer," *IEEE Journal of Solid-State Circuits*, vol. 43, no. 4, pp. 805–814, 2008.

[2] M. Park and M. H. Perrott, "A 78 dB SNDR 87 mW 20 MHz Bandwidth Continuous-Time $\Delta\Sigma$ ADC With VCO-Based Integrator and Quantizer Implemented in 0.13μm CMOS," *IEEE Journal of Solid-State Circuits*, vol. 44, no. 12, pp. 3344–3358, 2009.

[3] A. Babaie-Fishani and P. Rombouts, "A mostly digital VCO-based CT-SDM with third-order noise shaping," *IEEE Journal of Solid-State Circuits*, vol. 52, no. 8, pp. 2141–2153, 2017.

[4] S. Pavan, R. Schreier, and G. C. Temes, *Understanding delta-sigma data converters*. John Wiley & Sons, 2017.

[5] R. Pintelon, G. Vandersteen, J. Schoukens, and Y. Rolain, "Improved (non-) parametric identification of dynamic systems excited by periodic signals – the multivariate case," *Mechanical Systems and Signal Processing*, vol. 25, no. 8, pp. 2892–2922, 2011.

[6] R. Pintelon and J. Schoukens, *System Identification: a Frequency Domain Approach*. John Wiley & Sons, 2012.

[7] J. Schoukens, P. Guillaume, and R. Pintelon, "Design of multisine excitations," in *Control 1991. Control'91., International Conference on*. IET, 1991, pp. 638–643.

[8] G. H. Golub and C. F. Van Loan, *Matrix computations*. JHU Press, 2012, vol. 3.

[9] C. L. Phillips and H. T. Nagle, *Digital control system analysis and design*. Prentice Hall Press, 2007.

[10] A. Nuttall, "Some windows with very good sidelobe behavior," *IEEE Transactions on Acoustics, Speech, and Signal Processing*, vol. 29, no. 1, pp. 84–91, 1981.