Methodologies to determine b-term coefficients revisited

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Abstract

The accuracy of the longitudinal diffusion term (b-term) plays a vital role in the study of mass transfer mechanisms in high performance liquid chromatography (HPLC). In this study, three commonly used methodologies (peak parking; fitting of an experimental van Deemter curve; and the so-called dynamic method) for the determination of the b-term constant were investigated in detail. The three methods were compared based on their mutual agreement, the intra- and inter-day variation of the obtained values and the time required to measure them. Whereas the dynamic method was found to be plagued by impractically long waiting times and concomitant baseline variations compromising accurate measurements of the band broadening, the two other methods lead to very similar b-values, i.e., well within the 1% RSD inter-day variation typically marking both methods in the present study.

The best way to study the agreement of the peak parking and plate height fitting method is in a plot of $h \cdot v$ versus $v$, providing a much better zoom on the b-term region of the van Deemter curve than the customarily employed $h$ versus $v$-curve and hence allowing to identify any anomalous measurement values (usually related to measurements with a long experimentation time). Verifying the mutual agreement between both methods is proposed here as an additional accuracy check of the obtained data.

Keywords: longitudinal diffusion; b-term coefficient; peak parking; plate height models; dynamic method.
1. Introduction

In the past decades, column technology has evolved rapidly to meet the more challenging requirements of liquid chromatography users. Higher column efficiencies, better sample resolution and a far shorter analysis time are important requirements in modern day applications of chromatography. This has resulted in a significant progress in the design, manufacturing, and packing of narrow bore and short chromatographic columns with various types of packing materials. Columns packed with sub-2 µm or even nearly 1 µm particles and with lengths between 50 and 150 mm are nowadays commercially available [1–4]. These new column technologies have opened the way to faster and more efficient chromatographic separations. The resulting decrease in column permeability has been dealt with by the development of ultra-high pressure liquid chromatography equipment (UHPLC) able to operate at pressures up to 1500 bar [5–8]. As a compromise between column efficiency and permeability, column manufacturers have also successfully revived core-shell or superficially porous particles (sub-3 µm particle size), which provide a satisfactory sample loading capacity and allow fast separations and peak resolutions comparable with those of sub-2 µm fully porous particles without producing excessive backpressures [9,10].

The primary mechanical quality of a column is its efficiency, represented by the height equivalent to a theoretical plate height (HETP or H), which is mainly controlled by three independent and additive contributions: the dispersion due to the mobile phase velocity inequalities in the packed bed (a-term in the van Deemter equation), the dispersion due to the longitudinal diffusion (b-term in the van Deemter equation), and the resistance to the mass transfer of the solute from the moving mobile zone to the stationary phase zone and vice versa (c-term in the van Deemter equation) [11]. The present study focuses on the longitudinal diffusion (b-term), representing the sole source of dispersion which continues to go on when the flow is switched off. It encompasses the combined effect of the molecular species diffusion in the mobile zone (= zone outside particles) and in the stagnant mobile phase liquid inside the particles.
with the diffusion they experience when being in the retained state. Variations in its value mainly depend on the latter two, commonly combined under the term intra-particle diffusion [12,13].

An exact knowledge of the b-term is important to study the packing quality of columns (with the b- and c-term known, these contributions can be subtracted, leaving only the a-term dispersion). The re-introduction of core-shell particles has also demonstrated that the contribution of the b-term to band broadening is more important than was previously assumed [14]. Given its strong dependency on the intra-particle diffusion, an exact determination of the b-term constant, and more precisely its main constituting factor \(D_{\text{eff}}\), see Eq. (3) further on), furthermore also allows to produce a good estimate of the intra-particle diffusion coefficient \(D_{\text{part}}\), which in turn needs to be known to calculate the stationary zone contribution to the c-term dispersion [14–19].

The b-term constant can be determined in the absence or in the presence of a flow [20]. The most obvious approach (given the definition of the b-term dispersion as the single remaining source of axial band broadening in the absence of a flow) is the measurement in the absence of a flow (static approach), most commonly referred to as peak parking [18–21] and originally introduced by Giddings and Knox [22]. The dynamic method (i.e., in the presence of a flow) is performed by recording the peak profile of a compound at the lowest possible flow rate. This approach is generally justified by the consideration that the contributions of the a- and c-term band broadening become vanishingly small at very low flow rates and can hence anyhow mostly be neglected such that it is not really necessary to switch off the flow completely. In a third approach, recently adopted by McCalley et al. [16], the somewhat cumbersome process of selecting an appropriately low flow rate has been replaced by determining the b-term constant by fitting experimentally recorded plate heights of a compound over a whole range of velocities using one of the many existing plate height models (i.e., van Deemter equation, Giddings equation, Knox equation) [15–17].
In the present study, the above mentioned three different b-term determination approaches have been investigated in detail using a large-volume Zorbax Eclipse Plus C18 column (4.6 × 250 mm, 5 µm), and acetophenone and 3’-methylacetophenone as test molecules. The mutual agreement, the intra- and inter-day variation and the time required to perform each of these approaches are evaluated and compared. The goal is to help the chromatographic community to choose an adequate b-term determination method based on their necessities (cf. the compromise between accuracy and precision versus the available time).

Although we opted to focus on the interstitial velocity-based b-term constant (to be used in combination with the interstitial velocity \( u_i \)), it goes without saying that all observations and conclusions also directly apply to the unretained marker velocity-based b-term constant (to be used in combination with the unretained marker velocity \( u_0 \)). Given the definition of both velocities [23], the ratio between both b-term constants is simply equal to the ratio of the total over the external porosity of the bed (\( u_i/u_0 = \varepsilon_T/\varepsilon \)).

2. Experimental

2.1. Chemicals and Columns

HPLC grade acetonitrile (ACN) and tetrahydrofuran (THF) were purchased from Fisher Chemicals (Merelbeke, Belgium). Milli-Q water was prepared in the lab using a Milli-Q gradient water purification system from Millipore (Bedford, MA, USA). Acetophenone (99% pure) and 3’-methylacetophenone (98% pure) were obtained from Sigma-Aldrich (Steinheim, Germany). A set of 12 polystyrene standards with molecular weights ranging between 500 and 2,000,000 Da were purchased from Sigma-Aldrich (Bornem, Belgium). A Zorbax Eclipse Plus C18 column (4.6 × 250 mm, 5 µm) was purchased from Agilent Technologies (Diegem, Belgium).

2.2. Apparatus
All band-broadening experiments, molecular diffusion coefficient measurements ($D_m$) and inverse size exclusion chromatography (ISEC) experiments were performed on an Agilent 1290 UHPLC system (Agilent Technologies, Waldbronn, Germany) with a pressure limit of 1200 bar (further referred to as “instrument A” in the text). The instrument consisted of a quaternary pump, autosampler and diode array detector (DAD) with a flow cell of 1 µL. Chemstation software (Agilent Technologies) was used for instrument control, data acquisition and processing. The overall system volume was experimentally determined to be 13.9 µL. The injection volume was 1 µL and the sampling rate was varied between 20 and 40 Hz, depending on the flow rate.

To measure molecular diffusion coefficients ($D_m$), PEEK tubing (vendor specifications: 0.20 inch × 50 feet) with a measured length of 1532 cm and a calibrated internal diameter ($d_t$) of 0.051709 cm was purchased from GRACE (Columbia, MD, USA). The tubing was coiled into a diameter ($d_{coil}$) of 24 cm for temperature control and calibrated by weighing the amount of water contained within it. A thermostatted water bath JULABO (Seelbach, Germany) was used to maintain the temperature at 30°C.

Peak parking experiments were performed on the previously described Agilent 1290 UHPLC system and additionally on an Ultimate 3000 HPLC system from Dionex Softron GmbH (further referred to as “instrument B” in the text) equipped with a high-pressure pump (LPG-3400A), autosampler (WPS-3000SL) and UV/VIS variable wavelength detector (VWD-3400) with a flow cell of 11 µL. The overall system volume was experimentally determined to be 20 µL. The sampling rate was set at 40 Hz. Chromeleon software version 6.80 (Thermo Scientific) was used for data acquisition and instrument control. The injection volume used for the peak parking experiments was 1 µL.

For all band broadening and peak parking experiments, the column temperature was kept constant at 30°C by using a Spark Mistral oven (Emmen, Netherlands). All
experiments were conducted using two pieces of Viper tubing with an I.D. of 75 µm and length of 550 mm (Thermo Scientific) between the injector and the inlet of the column, and the outlet of the column and the detector, respectively. For the ISEC experiments, the detection wavelength was set at 225 nm, for the other measurements, it was set at 254 nm.

All data were analyzed using the method of moments and at half the peak height, using the instruments’ software. The latter data are shown in the supporting information.

2.3. Methodology

2.3.1. Sample and mobile phase preparation

Acetophenone and 3’-methylacetophenone were selected as the test molecules, since they were neutral and well-retained under the investigated conditions. For the band broadening and peak parking experiments, stock solutions of acetophenone and 3’-methylacetophenone were prepared in ACN, in a concentration of 3000 ppm and 4000 ppm, respectively. Fresh samples (acetophenone 300 ppm, 3’-methylacetophenone 320 ppm) were prepared daily by mixing and diluting stock solutions in the mobile phase. A set of twelve polystyrene standards (MW=500; 2,000; 3,000; 10,000; 20,000; 30,000; 70,000; 150,000; 300,000; 700,000; 1,000,000; 2,000,000) were used to perform ISEC experiments. Each standard was dissolved in THF at a concentration of 1 mg/mL.

For all plate height, peak parking and \( D_m \) measurements, the same batch of mobile phase was used with a composition of ACN/H\(_2\)O (45/55, v/v) leading to a zone retention factor of \( k'' \sim 2.8 \) for acetophenone and \( k'' \sim 4.7 \) for 3’-methylacetopheone. Figure 1 shows a representative chromatogram that was obtained at a flow rate of 1 mL/min, displaying narrow peaks with excellent peak shapes. Tetrahydrofuran was used as the mobile phase to perform the ISEC experiments.

2.3.2. Peak parking method
In this method, the analyte of interest was injected into the column at a flow rate of 0.5 mL/min. When the compound migrated approximately half way down the column, the flow was abruptly arrested for a given time ($t_{park}= 1$ min, 16 min, 31 min, 46 min, 61 min and 91 min). During this “parking time”, the analyte could diffuse freely under static conditions. Afterwards, the flow was resumed and the analyte peak eluted from the column towards the detector. These measurements were repeated four times for each parking time.

According to Einstein’s diffusion equation, the plot of the obtained peak variances ($\sigma^2_x$) of the sample band is a linear function of the applied peak parking time ($t_{park}$) [21,24]:

$$\sigma^2_x = 2 \cdot D_{eff} \cdot t_{park}$$  \hspace{1cm} (1)

The slope of this plot hence directly provides the value of the effective diffusion coefficient ($D_{eff}$) [13,14]. Bulk diffusion coefficients were measured at 30°C using the open tubular Taylor-Aris method wherein the variance ($\sigma^2$) and elution time (t) of a peak were measured after elution through a long, coiled capillary with diameter $d_t$ at a low flow rate (0.1 mL/min) [25]:

$$D_m = \frac{d^2 \cdot t}{96 \cdot \sigma^2_t}$$  \hspace{1cm} (2)

A low flow rate was applied to avoid secondary flow in the coiled capillary. The thus obtained molecular diffusion coefficients were $D_m= 1.19 \times 10^{-9}$ m$^2$/s for acetophenone and $D_m= 1.08 \times 10^{-9}$ m$^2$/s for 3’-methylacetophenone.

The reduced b-term coefficient was subsequently calculated from the experimentally determined values of $D_{eff}$ and $D_m$ as follows [14]:

$$b = 2 \frac{D_{eff}}{D_m} (1 + k'')$$  \hspace{1cm} (3)

Wherein the zone retention factor ($k''$) is defined as (see Eq. 9 for $u_i$):

$$k'' = \frac{t_{R} \cdot L}{u_i}$$  \hspace{1cm} (4)
To test the inter-day variation of the peak parking method, the peak parking measurements were repeated for each studied compound on two consecutive days using the same HPLC system, and on a third day using a completely different HPLC system (see § 2.2).

2.3.3. Plate height curve fitting method

2.3.3.1. Measurement of the plate height data

Plate height data were measured at 21 different velocities (ranging between 0.02 and 1.3 mL/min) in isocratic mode, in order to generate sufficient data points for a precise curve fitting. Measurements were performed in triplicate, by performing one injection at each flow rate starting from the lowest flow rate until all flow rates were evaluated, and then repeating this cycle two more times. During the plate height measurements, the accuracy of each flow rate was monitored by collecting the mobile phase at the outlet of the detector for a certain time. During the whole series of experiments, the column temperature was carefully monitored as well with a thermocouple (Fluke, Washington, United States) The observed temperature never deviated more than 0.2°C from the set temperature.

Peak variance ($\sigma^2$) and elution time ($t_R$) values were corrected for the system band broadening ($\sigma^2_{sys}$) and dead time ($t_{sys}$) [26]. The latter were determined by replacing the column with a zero dead-volume connector under the same experimental conditions as for the plate height measurements [27]. The system variance was less than 1.2% of the total variance of acetophenone, and less than 0.6% of the total variance of 3'-methyl acetophenone under all investigated conditions.

Plate counts (N) and plate heights (H) were subsequently calculated as follows:

$$N_{col} = \frac{(t_{R, total} - t_{R, sys})^2}{\sigma_{total}^2 - \sigma_{sys}^2}$$ \hspace{1cm} (5)

$$H_{col} = \frac{L}{N_{col}}$$ \hspace{1cm} (6)
Where $L$ is the column length, the subscript “total” refers to the experimentally measured variance and analysis time and “col” refers to the pure column efficiency obtained after system correction.

Reduced plate heights ($h$) and reduced interstitial velocities ($v_i$) were calculated as:

$$h = \frac{H_{col}}{d_p}$$  \hspace{1cm} (7)

$$v_i = \frac{u_i \times d_p}{d_m}$$  \hspace{1cm} (8)

With $d_p$ the column particle size and $u_i$ the interstitial velocity, determined as:

$$u_i = \frac{F}{\varepsilon_e \pi r^2}$$  \hspace{1cm} (9)

Where $\varepsilon_e$ is the external porosity and $r$ is the column radius.

The external porosity ($\varepsilon_e$) was measured experimentally by inverse size exclusion chromatography (ISEC) using a set of twelve polystyrene standards [28]. The flow rate was set at 0.4 mL/min, and injection volumes were 1 µL. Each injection was performed in triplicate and the obtained retention volumes averaged. Retention volumes were corrected for the extra-column volume of the system. The elution volumes of the polystyrene standards were subsequently plotted against the cubic root of their molecular weights ($MW^{1/3}$). The external porosity was derived by extrapolating the exclusion branches of the ISEC plots to $MW^{1/3} = 0$ [29] and was determined to be 41.74% in this way.

2.3.3.2. Fitting of the plate height data

The b-term coefficients were determined by fitting the experimentally obtained plate height data to the following plate height models:

- van Deemter equation: $h = a + \frac{b}{v_i} + cv_i$  \hspace{1cm} (10)

- Knox equation: $h = av_i^{1/3} + \frac{b}{v_i} + cv_i$  \hspace{1cm} (11)

- free n-Knox equation: $h = av_i^n + \frac{b}{v_i} + cv_i$  \hspace{1cm} (12)
The fitting procedure was performed using the solver function of Microsoft Excel, using the GRG Nonlinear Solving engine (constraint precision: 0.000001, convergence: 0.0001 and population size: 100).

2.3.4. Dynamic method

In this method, the b-term coefficients were calculated based on the experimental plate height data obtained at very low flow rates, based on the assumption that the a- and c-terms are very small at these flow rates and therefore negligible:

$h \approx h_b = b / v \Rightarrow b \approx h \times v$  \hspace{1cm} (14)

3. Results and Discussion

3.1. Peak parking method

Peak parking measurements were first performed on instrument A by determining peak variances ($\sigma^2_x$) at specific parking times. The obtained average values of $\Delta\sigma^2_x$ (out of four measurements per parking time) and their standard deviations shown as error bars as a function of the parking time are shown in Figure 2 (diamonds). Note that the standard deviations are so small, that they are hardly visible, indicating that the intra-day variability is negligible. To omit the effects of any possible variation in parking time, the data were plotted as a function of the actual parking time (calculated by subtracting the retention time of the peak when no parking time is applied from the retention time that is obtained when a certain parking time is applied). Note that the variances are reported as $\Delta\sigma^2_x$-values, obtained by subtracting the value for $\sigma^2_x$ at $t_{park}=1$ min from the variances at all subsequent parking times. From the slopes of the straight line relationship in Fig. 2, the effective diffusion coefficient $D_{eff}$ was calculated according to Eq. (1), after which the b-term coefficient was calculated according to Eq. (3).
To assess the inter-day variability of the peak parking method, the same experiment was
subsequently performed on the same instrument (instrument A), but on a second day,
while the same experiment was also repeated on a third day, but now using a different
HPLC system (instrument B), with a different system volume. The corresponding
curves of $\Delta \sigma^2_{x}$ versus parking time have also been added to Figure 2 (squares and
triangles). The average b-term values obtained from the slopes of the curves of $\Delta \sigma^2_{x}$
versus $t_{park}$ by performing peak parking experiments on three different days were
calculated to be $b = 5.25 \pm 0.06$ for acetophenone, and $b = 6.75 \pm 0.03$ for 3'-methyl
acetophenone. Considering these data were measured on different days, using different
instrumentation, the observed inter-day variability of maximum 1.1% can be considered
to be very good.

To obtain a maximal insight in the generated data, the data in Figure 2 were
subsequently replotted in Figure 3 by plotting $\Delta \sigma^2_{x} \frac{(1+k'')}{(D_m t_{park})}$ as a function of
time to obtain a more zoomed-in view on the data. This approach is followed because it
can be deduced from Eqs. (1) and (3) that the b-term values obtained at each individual
calling time can also be directly calculated as follows:

$$b = \frac{\Delta \sigma^2_{x}}{t_{park}} \frac{(1+k'')}{(D_m)} \quad (15)$$

The resulting b-term values (per individual parking time) are shown in Figure 3. A
rather complex pattern can be observed, with some random variations between the
different days (and instruments). The intra-day variability of the values obtained for the
individual times (for four consecutive injections on the same day, represented by the
error bars) was maximum 1.2%, while the inter-day variability (by performing the same
experiments on three consecutive days and on two different instruments) was maximum
2.5%. The inter-day variability of the average values of the individual parking time
values was much lower, resp. 0.6 and 0.7% for the two components (see Table 1).
To investigate some of the observed variations in more detail, the raw chromatograms obtained at different parking times were studied in more detail, but no anomalies or systematic changes could be found. All peaks were symmetrical at all parking times and all displayed a sufficiently large signal-to-noise ratio. This was confirmed by the fact that experiments performed at longer analysis times did not result in larger error bars, as can be deduced from Figure 3. Another potential source for the observed variation is the temperature at which the experiments were performed. However, all experiments were performed in a thermostatted oven compartment at 30.0°C of which the temperature was closely monitored and never deviated more than \( \pm 0.2°C \). A clear explanation for the observed variation, which was anyhow limited, could therefore not be found. Note that similar variations were observed when the data were analyzed at half the peak heights, as can be seen in Figure S-2 in the Supporting Information.

Figure 3 also clearly shows that the variation obtained when performing the peak parking experiment on two different instruments is similar to or smaller than the variation obtained when performing the peak parking experiment on the same instrument, but on a different day. It can hence be concluded that the instruments themselves did not have any significant effect on the obtained b-term values. Similar observations were made when the data were analyzed at half the peak height (see Figure S-2 in the Supporting Information). The good agreement in slope values between the data analyzed at half the peak height and using the method of moments, moreover suggests a high symmetry of the obtained peaks.

### 3.2. Plate height curve fitting method

#### 3.2.1. Measurement of the plate height data

Plate height curves were measured in triplicate by performing one injection per flow rate, from the lowest flow rate (set at 0.02 mL/min for practical reasons, see also further below) to the highest possible flow rate (limited by the maximum pressure of the column), and then repeating this cycle two additional times to investigate the maximum
possible variation in plate height. Since it took an entire day to measure one plate height
curve, and each plate height curve was hence recorded at a different day, these
measurements also immediately resulted in an assessment of the inter-day variability.

Figure 4 shows the average values of the plate heights that were obtained in this way
for acetophenone (Figure 4a) and 3’-methylacetophenone (Figure 4b), respectively. The
error bars represent the inter-day variability calculated as the RSD on the data. Figure
S-3 in the Supporting Information shows the equivalent of this figure, but now relating
to the peak width at half height. The accuracy of the flow rate was investigated by
collecting the volume of mobile phase eluting from the column for the duration of the
experiment at a specific flow rate (see section 2.3.3.1). The deviation between the set
and measured flow rates was below 1% for all flow rates.

At the lowest flow rate of 0.02 mL/min, the inter-day variability on the plate height
values amounted up to 4.8% for acetophenone and 11.6% for 3’-methylacetophenone.
These high RSD values can most probably be directly related to the high degree of
baseline drifting that was observed at this low flow rate (data not shown), which in turn
can be directly explained by the very long analysis times (order of 10h!) associated with
the measurement at this low flow rate. Baseline drifting can result in large errors in
peak integration and peak height read-out and can hence strongly influence the accuracy
of the measured plate heights. The plate height values obtained via the width at half the
peak height showed a similar significant inter-day variability at the lowest flow rate,
albeit slightly lower: 3.7% and 10.0% for acetophenone and 3’-methylacetophenone,
respectively (see Figure S-3). Some baseline drifting was also observed at the other low
flow rates of 0.025 and 0.030 mL/min, but the extent of this drifting was much smaller
and only resulted in a plate height variation of 0.8% for acetophenone and 1.6% for 3’-
methylacetophenone at 0.025 mL/min when analyzed using the method of moments.
For all other flow rates, the variation in plate height values remained below 1% for both
compounds. These observations can also be directly made from the error bars added to Figure 4 (and Figure S-3).

Baseline drift over the long measurement times associated with the very low flow rates applied here are certainly not uncommon. One potential cause for this baseline drift could be excluded as the column temperature was monitored carefully during all plate height measurements and never deviated more than 0.2°C from the set temperature. Other precautions (such as the prevention of the evaporation of solvents, manual premixing and degassing of the mobile phase) were taken as well. A possible explanation for the baseline drift could be small variations in the temperature of the flow cell during the course of the longer analytical runs (at the lowest flow rates), since the option to continuously heat the flow cell was not available.

3.2.2. Fitting of the plate height data

The measured plate heights were subsequently reduced into dimensionless coordinates using Eqs. (7-8) and fitted to four plate height equations: the van Deemter, Knox, free n-Knox and Giddings equation (see 2.3.3.2). The result is shown in Figure 5a for acetophenone (analyzed using the method of moments). To assess which plate height model fitted the experimentally obtained data best in the region of low velocities, the original plate height data were replotted in such a way that a more zoomed-in view of the data in this region was obtained (Fig. 5b). This was done by transforming the y-axis from h to h·v_i. As can be verified from any of the plate height models given in Eqs. (10-13), this transformation leads to a curve that should directly converge to the value of b when v_i goes to zero. The h·v_i-plot can moreover be very helpful to assess the quality of the measured data and their fitting in the b-term regime, while the customarily used h-plot does not give any clear detail. For the considered data set, the h·v_i-plot in Figure 5b for example readily reveals that the first two data points (at the lowest velocities) are not in line with the other points at higher velocities and can hence be considered as measurement outliers or artefacts, most probably related to the baseline drift problem.
occurring at these low flow rates as discussed above. Fitting the plate height data including the first two data points lead to very poor quality fits with clearly inconsistent parameter values (data not shown). However, by omitting the first two data points from the fitting, as is shown in Figure 5b, excellent fits are obtained. Figure 5b also shows that very little difference in fitting quality is obtained when using either the Knox, the free n-Knox or the Giddings model. Given Knox himself eventually preferred the free n-Knox model (with n rather in the order of 0.5 to 1) over the more customarily used n=1/3-variant [30], and given the Knox-model can be seen as a practically useable approximation of the multi-scale Giddings model [22], we continued the rest of the study with the free n-Knox model. Fitting the data with the other models, the effect on the obtained b-values was however always very small (maximum variation in b-term value of 1%)

3.3. Dynamic method

The dynamic method consists of calculating the b-term coefficient from the plate height value measured at the lowest possible flow rate using Eq. (14). According to the specifications of the manufacturer, the instrument A used in this study can reach a minimum flow rate of 0.01 mL/min with great flow precision. In the current study, using well-retained compounds (k"= 4.66 for 3'-methylacetophenone), a flow rate of 0.01 mL/min would lead to an impractically large elution time, increasing the chance of baseline drifting. Therefore, a flow rate of 0.02 mL/min was chosen as the minimum flow rate.

As shown in Table 1 (bottom row), the b-term values obtained via Eq. (14) at this low flow rate are far off from all the other b-values, and furthermore display a very high inter-day variability. This should not be surprising, giving the flow rate used for the dynamic method is the same as the lowest flow rate in Figure 5, and hence suffers from the same baseline drift problem already discussed in the previous section.
Considering the accuracy of the method, yet another problem arises. Using any of the models represented by Eqs. (10-13) with typical values for a, b, c, d and n, it can be verified that the reduced velocities below which the a- and c-term contributions become less than 1% of the total plate height is of the order of \( \nu_i = 0.05 \text{ - } 0.1 \). This is even less than the reduced velocity related to the lowest flow rate we could adopt in the present study. Taking into consideration the column particle size (\( d_p = 5 \text{ µm} \)), column I.D. (4.6 mm), external porosity (\( \varepsilon_e = 41.7\% \)), and molecular diffusion coefficients (\( D_{m, \text{acetophenone}} = 1.19 \times 10^{-9} \text{ m}^2/\text{s} \) and \( D_{m, \text{3'-methyl acetophenone}} = 1.08 \times 10^{-9} \text{ m}^2/\text{s} \)), the reduced interstitial velocities corresponding to the 0.02 mL/min flow rate are on the order of \( \nu_i = 0.2 \) for both components, i.e. well above the \( \nu_i = 0.05 \text{ - } 0.1 \) criterion, implying the dynamic method will never reach accurate values of the b-term coefficient.

3.4. Comparison of the different methods for the determination of the b-term coefficient

Figure 6 gives an overview of the different methods in a \( h \cdot \nu_i \)-plot. The open symbols added at \( \nu_i = 0 \) represent the values obtained with the peak parking method for the individual parking times (as shown in Figure 3). The two horizontal lines represent the average of these points (dashed line) and that derived from the slope in the \( \Delta \sigma_x \) versus \( t_{park} \)-plots (solid line). The b-value corresponding to the lowest reduced velocity point (somewhere around \( \nu_i = 0.2 \)) also represents the value obtained with the dynamic method.

As can be noted, the \( h \cdot \nu_i \)-plots in Figure 6 are ideally suited to show that there is a very good agreement between the fitting method (at least when omitting the data points that are clearly off the trend of the other data points in the van Deemter curve; in this case the first two data points for acetophenone, and the first three data points for 3'-methyl acetophenone) and the peak parking method. The difference in b obtained via the fitting method and any of the two horizontal lines obtained via peak parking is not larger than
Similar observations are also made for the data analyzed at half the peak height as can be seen in Figure S-5 in the Supporting Information.

Figure 6 also readily shows that the dynamic method can be discarded as a useful method. Similar conclusions can be drawn from Table 1, summarizing the b-term values obtained using three different methods of determination, together with their inter-day variability (hence the variation in b-term coefficient obtained by performing experiments on three different days). As can be noted, the maximum variation in b-term obtained by fitting the plate height data to the free n-Knox model and using the slope of the peak parking curves is 0.4% for acetophenone and 1.7% for 3’-methylacetophenone. Comparing these values to the average b-term values obtained for the individual parking times, this variation is maximum 1.1% for acetophenone and 0.3% for 3’-methylacetophenone.

Also indicated in Table 1 is the total time required for each of the methods. The calculated required time includes the actual run time (retention time × 1.125) and processing time (1 min per chromatogram). Considering the inter-day variability obtained by performing experiments on three consecutive days was around or below 1% for most experiments, it should suffice to perform each experiment only once to obtain b-term data with sufficient reliability. Therefore, the required measurement time was calculated assuming each individual experiment was only carried out once. For the b-term values determined at individual peak parking times, the time to perform an experiment with \( t_{\text{park}} = 1 \) min was also taken into consideration.

From Table 1, it can be deduced that the dynamic method can be performed relatively fast (determination time comparable to or even smaller than the entire time required to perform peak parking). However, as was already clear from Figure 6, the dynamic method will never lead to accurate values of the b-term coefficient. Peak parking experiments conducted at a single peak parking time can also already produce accurate
values of the b-term in very short times. Comparing the b-values obtained from the slopes of the curves of $\Delta \sigma^2_{x}$ versus $t_{park}$ with the b-values obtained at the individual parking times, the maximum deviation is around 2%.

The time required to record and analyze all plate height data required for the curve fitting is 4-5 fold larger compared to the other methods. When performing column performance studies, however, information on the a- and c-term is usually also desired, in which case plate height data covering a larger range of flow velocities are anyhow required.

3.5. Implications for the determination of the a-term contribution

Considering the inter-day variability data reported in Table 1, it is now interesting to assess what the implications of a certain variation in b-term value will be for the further determination of column performance parameters. From the general plate height model:

$$h = h_{eddy} + \frac{2 \cdot D_{\text{eff}}}{v_{l} \cdot D_{m}} (1 + k'') \frac{\varepsilon_{e}}{1 - \varepsilon_{e}} + \frac{2 \cdot k'^2}{\alpha \cdot (1 + k'')^2 \cdot 1 - \varepsilon_{e} \cdot Sh_{m}} + \frac{2 \cdot k'^2}{\alpha \cdot (1 + k'')^2 \cdot Sh_{\text{part}}} \frac{v_{l}}{D_{\text{part}} \cdot D_{m}}$$

wherein $D_{\text{part}}$ is the intra-particle diffusion coefficient, $\alpha$ a geometrical constant and $Sh_{m}$ and $Sh_{\text{part}}$ are the Sherwood numbers relating to the mobile and the intra-particle zone, respectively, it is clear that the determination of the b-term coefficient (or $D_{\text{eff}}$) will have implications on different levels.

The first level is of course the determination of the contribution of longitudinal diffusion itself. But also the contribution of the mass transfer resistance in the stationary zone will be affected by the b-value because $D_{\text{part}}$ directly depends on $D_{\text{eff}}$ (see Eq. 3 for relation between $b$ and $D_{\text{eff}}$) according to following equations:

$$D_{\text{eff}} = \frac{1}{D_{m}} \frac{1 + 2 \beta_{1} (1 - \varepsilon_{e}) - \varepsilon_{e} \zeta_{2} \beta_{1}^2}{\varepsilon_{e} (1 + k'') \cdot 1 - \varepsilon_{e} (1 + k'') \cdot 1 - \varepsilon_{e} \zeta_{2} \beta_{1}^2}$$

Wherein, $\zeta_{2}$ is a geometrical three-point parameter [31], $\beta_{1}$ is the polarizability constant:

$$\beta_{1} = \frac{\alpha_{\text{part}}^{-1}}{\alpha_{\text{part}} + 2}$$

and $\alpha_{\text{part}}$ is the relative particle permeability:
Following Eqs. (17-19), it can be calculated that a 1% deviation in b-term (and hence a 1% deviation in $D_{\text{eff}}/D_m$-value) will lead to a 1.7% variation in $D_{\text{part}}/D_m$-value and hence a 1.7% variation in $h_{cs}$-value. For higher variations in b-term values (cfr. Table 1), this variation in $h_{cs}$-term increases to 3.4% (2% variation in b-term) and 5.1% (3% variation in b-term).

Finally, also the determination of the eddy dispersion term (the first term in Eq. (16)) will be largely affected by the determination of the b-term value. Customarily, the value of the a-term is determined by subtracting the b- and c-term contributions from the overall measured plate height value.

Figure 7 shows the eddy dispersion terms that were calculated in this way for acetophenone and 3’-methylacetophenone. Note that the individual contributions of the b-term, $h_{cs}$-term and $h_{cm}$-term were calculated according to Eq. (16), with $\alpha = 6$, $S_{\text{h}}_{\text{par}} = 10$ and $S_{h_m} = \frac{1.09}{\varepsilon_e} \cdot (\varepsilon_e \cdot \nu_i)^{\frac{1}{3}}$ [32]. The dashed lines indicate the a-term contribution that would be obtained for a 1% and 2.5% deviation of the b-term, showing the huge impact such seemingly small variations on the b-term constant have on the observed a-term, especially in the low velocity range. For a 1% deviation in b-term, a variation in a-term ranging between 133% at $\nu_i = 0.20$ and 11% at $\nu_i = 1.2$ is obtained for acetophenone. Only when the reduced velocity exceeds $\nu_i = 6.0$, the variation in a-term becomes smaller than 1%. For 3’-methylacetophenone, the observed variation in a-term is slightly lower, ranging between 10 and 50% for reduced velocities of $\nu_i = 0.22$ and 1.6. For a deviation of 2.5% in b-term these numbers are higher, a maximum variation in a-term of 330% (acetophenone) and 125% (3’-methylacetophenone) are obtained, and the variation in a-term only becomes smaller than 1% once the velocity exceeds $\nu_i = 12$ (acetophenone) and $\nu_i = 14.5$ (3’-methylacetophenone).
4. Conclusions

The three main methodologies (peak parking, dynamic method, plate height curve fitting) for the determination of the longitudinal diffusion or b-term constant have been revisited on a large, 5 μm particle column, and using acetophenone and 3’-methylacetophenone as test molecules.

Carefully measuring the raw data, the peak parking and the plate height fitting method were found to be in excellent agreement. The difference between the two methods was less than 0.7%, i.e., smaller than the inter-day variability of the individual methods. The fitting method takes longer to execute, but when one is anyhow measuring a full van Deemter curve, the fitting method can produce an accurate b-term value without any additional effort.

For those analysts willing to spend maximum effort to pursue a maximal accuracy, it is proposed here to combine both methods and use the type of \( h.v_i \)-plots shown in Figure 5b and Figure 6 to study their convergence and eliminate anomalous measurement points. For what concerns the peak parking method, possible measurement anomalies can best be detected using the type of \( \Delta \sigma^2_x(1+k''')(D_m.t_{park}) \)-plots shown in Figure 3, whose y-axis furthermore provides a direct read-out of b. The plot also shows that good measurements of the b-constant can be obtained measuring at only one parking time (in fact two because also \( t=1 \) min needs to be determined).

The dynamic method needs to be carried out at such low flow rates that it leads to impractical analysis times, which furthermore increase the likelihood of severe baseline drifts, which in turn leads to totally unreliable measurements. The plate height fitting method is also plagued by this problem if one tries to include velocity data points that are too small. This leads to the somewhat contradictory conclusion that the velocities used in the plate height curve should not be taken too small, whereas the b-constant is in theory best approached when the velocity tends to zero (or is zero).
Overall, it seems difficult to determine the b-term constant with an RSD below 1%. Although this precision in itself should be considered to be very good, the remaining uncertainty leads to a very high uncertainty on the precise course of the a-term velocity dependence. For the present column, the 1% uncertainty on b leads to an uncertainty on the a-term contribution above 100% at $v_i = 0.2$ and around 10% at $v_i = 1$. Overall, the data show that even with an extreme accuracy of the b-term measurement, it is intrinsically nearly impossible to determine the exact velocity dependence of the a-term contribution for all reduced velocities below $v_i = 5$. 
5. References


6. Acknowledgements

Agilent Technologies is kindly acknowledged for the gift of the Infinity 1290 system through a University Relations Grant. Huiying Song acknowledges KU Leuven for the post-doctoral mandate (PDM).
7. Figure Captions

**Figure 1:** Representative chromatogram of acetophenone (k”= 2.81) and 3’-methylacetophenone (k”= 4.66) obtained on a Zorbax Eclipse Plus C18 column (4.6 × 250 mm, dp = 5µm) at a flow rate of 1 mL/min. Mobile phase is ACN/H₂O (45/55, v/v).

**Figure 2:** Curves of Δσₓ² versus parking time (tpark) for (a) acetophenone and (b) 3’-methylacetophenone on (●) instrument A on day 1, (▲) instrument A on day 2, and (■) instrument B on day 3. Error bars indicate the standard variations obtained by performing the same experiment four times consecutively on the same day. Data analyzed using the method of moments. The equations of the regression curves best fitting the experimental data are also given.

**Figure 3:** Δσₓ²/(2·tpark)-data versus tpark representation of the data shown in Figure 2 for (a) acetophenone and (b) 3’-methylacetophenone on (●) instrument A on day 1, (▲) instrument A on day 2, and (■) instrument B on day 3. Error bars indicate the standard variations obtained by performing the same experiment four times consecutively on the same day. Connecting lines have been added with the sole purpose of guiding the eye.

**Figure 4:** Curves of plate height (H) versus interstitial velocity (ui) for (a) acetophenone and (b) 3’-methylacetophenone. The displayed data are the average values obtained for three measurements on three different days. Standard deviations on these values are shown as error bars. Data analyzed using the method of moments.

**Figure 5:** (a) Curves of h versus vi and (b) h·vi versus vi for acetophenone. The symbols indicate the raw plate height data, the lines are the fitted curves obtained by fitting the experimental data to one of following plate height models: ( ) van Deemter, ( ) Knox, (---) free Knox, ( ) Giddings.-

**Figure 6:** Curves of h·vi versus vi for (a) acetophenone and (b) 3’-methylacetophenone
for experimental plate height data analyzed via the method of moments. Experimental plate height data were fitted using the free Knox model by omitting the anomalous data points for fitting as explained in the text. The horizontal lines represent the b-term values obtained from the slope of the peak parking experiments ( ) and from the average of the individual peak parking data ( ). The b-term values obtained for the individual peak parking experiments are shown by the open symbols on the y-axis (◊ $t_{park}= 15$ min, ○ $t_{park}= 30$ min, □ $t_{park}= 45$ min, △ $t_{park}= 60$ min and × $t_{park}= 90$ min). To help interpret the precision, the double arrow on the y-axis indicates a 1% variation (RSD) in b-term value.

**Figure 7:** Curves of $h_a$ versus $v_i$ for (a) acetophenone (♦) and (b) 3’-methylacetophenone. The a-term contribution is calculated by subtracting the b-, $h_{cs}$- and $h_{cm}$- term from the overall plate height. The dashed lines indicate the a-terms that would be obtained when the b-term would vary with 1% (----) or 2.5% (---).
Figure 1

acetophenone $k''=2.81$

3'-methyl acetophenone $k''=4.66$
Figure 2

(a) $\Delta \sigma^2_x (m^2) = 1.629E-09x + 9.119E-09$

(b) $\Delta \sigma^2_x (m^2) = 1.287E-09x - 2.082E-08$
Figure 5

(a)

(b)
Figure 6

(a) $h \cdot v_i$

(b) $h \cdot v_i$

± 1%
Figure 7
### 3′-methylacetophenone

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### curve fitting*

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### dynamic method

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* curve fitting performed by omitting the first 3 data points.
Supporting Information

Methodologies to determine b-term coefficients revisited

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Abstract

The Supporting Information supplies a number of figures that were obtained by analyzing the data represented in the original manuscript at half the peak height.

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**Figure S-1:** Curves of $\Delta\sigma_x^2$ versus parking time ($t_{park}$) for (a) acetophenone and (b) 3'-methylacetophenone on (◊) an Infinity 1290 UHPLC instrument on day 1, (▲) an Infinity 1290 UHPLC instrument on day 2, and (■) an Ultimate 3000 HPLC instrument on day 3.

**Figure S-2:** Curves of $\Delta\sigma_x^2/(2 \cdot t_{park})$ versus $t_{park}$ for (a) acetophenone and (b) 3'-methylacetophenone on (◊) an Infinity 1290 UHPLC instrument on day 1, (▲) an Infinity 1290 UHPLC instrument on day 2, and (■) an Ultimate 3000 HPLC instrument on day 3.

**Figure S-3:** Curves of plate height ($H$) versus interstitial velocity ($u_i$) for (a) acetophenone and (b) 3'-methylacetophenone. The displayed data are the average values obtained for three measurements on three different days.

**Figure S-4:** Curves of $h \cdot v_i$ versus $v_i$ for acetophenone analyzed at half the peak height. The symbols indicate the raw plate height data, the lines are the fitted curves obtained by fitting the experimental data to one of following plate height models: ( ) van Deemter, ( ) Knox, ( ) free Knox, ( ) Giddings. The first two data points were omitted for the curve fitting.

**Figure S-5:** Curves of $h \cdot v_i$ versus $v_i$ for (a) acetophenone and (b) 3'-methylacetophenone for experimental plate height data analyzed at half the peak height. Experimental plate height data were fitted using the free n-Knox model by omitting the first two datapoints ( ). The horizontal lines represent the b-term values obtained

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from the slope of the peak parking experiments ( ) and from the average of the individual peak parking data ( ). The b-term values obtained for the individual peak parking experiments are shown by the open symbols on the y-axis (◊ \( t_{park} = 15 \) min, ○ \( t_{park} = 30 \) min, □ \( t_{park} = 45 \) min, △ \( t_{park} = 60 \) min and × \( t_{park} = 90 \) min).
**Figure S-1:** Curves of $\Delta \sigma_x^2$ versus parking time ($t_{park}$) for (a) acetophenone and (b) 3’-methylacetophenone on (♦) an Infinity 1290 UHPLC instrument on day 1, (▲) an Infinity 1290 UHPLC instrument on day 2, and (■) an Ultimate 3000 HPLC instrument on day 3. Error bars indicate the standard variations obtained by performing the same experiment four times consecutively. Data analyzed at half the peak height.
Figure S-2: Curves of $\Delta \sigma_x^2/(2 \cdot t_{park})$ versus $t_{park}$ for (a) acetophenone and (b) 3’-methylacetophenone on (♦) an Infinity 1290 UHPLC instrument on day 1, (▲) an Infinity 1290 UHPLC instrument on day 2, and (■) an Ultimate 3000 HPLC instrument on day 3. Error bars indicate the standard variations obtained by performing the same experiment four times consecutively. Data analyzed at half the peak height.
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