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Published in:
Journal of Quantitative Analysis in Sports

DOI:
10.1515/jqas-2020-0009

Publication date:
2021

License:
Unspecified

Document Version:
Final published version

Link to publication

Citation for published version (APA):
Blondeau, J. (2021). The influence of field size, goal size and number of players on the average number of goals scored per game in variants of football and hockey: the Pi-theorem applied to team sports. *Journal of Quantitative Analysis in Sports*, *17*(2), 145-154. https://doi.org/10.1515/jqas-2020-0009

Research article

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The influence of field size, goal size and number of players on the average number of goals scored per game in variants of football and hockey: the Pi-theorem applied to team sports

https://doi.org/10.1515/jqas-2020-0009
Received January 28, 2020; accepted August 25, 2020;
published online September 21, 2020

Abstract: In this paper, we investigate the correlation between the main physical characteristics of eight variants of football and hockey (such as field size, goal size, player velocity, ball velocity, player density, and game duration) and the resulting average numbers of goals scored per game. To do so, the Pi-theorem in physics is extended to sport science and a non-dimensional parameter of interest is defined. It is based on the ratio between the duration of the game and the order of magnitude of the time needed to cross the midfield, which depends on the average velocity of the ball and the players, the player density and the size of the goals. An excellent correlation is found between the proposed parameter and the average number of goals scored per game during recent international competitions. Using the derived correlation, the effect of any modification of the main characteristics of football and hockey (and their variants) on the scoring pace can be assessed. For instance, it can be predicted that decreasing the length of football fields by 20 m would raise the average number of goals scored to 3.6 (±0.6) per game, versus the 2.6 goals scored during the most recent men's World Cup.

Keywords: field size; football; goal size; goals; hockey.

1 Introduction

In team sports such as football and hockey, the field size, the goal size, the player velocity, the ball velocity, the number of players and of course the game duration have a direct influence on the average number of goals scored per game. This becomes obvious when comparing various forms of these sports. For instance, ice hockey players can potentially move much faster than field hockey players. They also play on a much smaller field. This could explain why they score more goals per game in average, despite the smaller goal size and the shorter game duration. Futsal players also score much more than football players, despite the smaller goal size and shorter game duration. A possible explanation is that their field is however much smaller, even (and especially) compared to the relative goal size reduction. These variants of football and hockey are all similar, but they are played at different scales. Questions about the relative influences of their key characteristics may arise from these considerations, for instance about how much the scoring pace would be affected by a modification of the field length or the goal size. Quantifying these relative influences could lead to ad hoc modifications of some characteristics of these sports, for instance to adjust the average number of goals scored per game to a desired level.

In this paper, we investigate the correlation between the main physical characteristics (i.e., field size, goal size, player density, player velocity, ball velocity, and game duration) and the number of goals scored per game in international competitions for eight variants of football and hockey. These team sports are indeed of great interest for such a study, as they are similar, and are both played in various forms (i.e., at various scales) for which statistical data can be retrieved at the international level. The following eight variants of hockey and football are considered in this study: Football (F), Futsal (Fs), Beach Soccer (BS), Field Hockey (FH), Indoor Hockey (InH), Hockey 5s (H5s), Ice Hockey (IcH), and Inline Hockey (IiH).

In all these sports, ad hoc modifications of the number of players, the field size, the goal size, the game duration and even the rules of game can be applied during training sessions. For instance, Small-Sided Games (SSG) aim at

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increasing the training intensity compared to full games, whilst subsequently developing specific physical and tactical skills (Halouani et al. 2014; Hammami et al. 2017; Sarmento et al. 2018). They are also called skill-based conditioning games (Hammami et al. 2017). In studies dedicated to SSG, the physiological response of the players is generally studied from a training perspective. The effect of SSG on fitness, sprinting, jumping, and agility can be assessed (Hammami et al. 2017). The most prevalent topic in SSG research is the effect of number of players. Fewer players significantly increase the physiological demand (Sarmento et al. 2018). Tactical aspects cover for instance the number of successful passes, blocks, dribbles, or interceptions (Sarmento et al. 2018). This study, however, is not concerned by the detailed physical and tactical performances of the individual players during training, but by the general correlation between the main physical parameters of eight team sports, including the representative velocities of the players as they run or skate, and the final scores observed in international competitions.

Our first objective is to find a non-dimensional parameter based on the basic characteristics of the eight studied sports that satisfyingly correlates with the average number of goals scored per game. This investigation is carried out as an extension of the Pi-theorem in physics which states that any physical phenomenon is ruled by a limited number of non-dimensional parameters. These governing non-dimensional numbers are generally based on a relative comparison of different time- or length-scales (see Section 2).

Our second objective is to use the derived correlation to predict the effects of some modifications of the basic characteristics of football and hockey on the expected number of goals scored per game if they were integrated in the official rules.

2 The Pi-theorem in physics

The Pi-theorem in physics, also called the Buckingham or the Vaschy-Buckingham theorem, is a result of dimensional analysis. It is based on the independence of physical correlations on the unit system used to express the various physical quantities at stake. It states that any functional equation for a physical quantity a depending on n dimensional parameters $a_1, \ldots a_n$:

$$a = f(a_1, a_2, \ldots, a_n) \tag{1}$$

can be reorganized to the form of the non-dimensional equation

$$\Pi = \varphi(\Pi_1, \Pi_2, \ldots, \Pi_{n-k}) \tag{2}$$

that contains $(n - k)$ non-dimensional variables (Yarin 2012), where k is the number of independent dimensions of the parameters of Eq. (1): for instance time, length, mass, etc. Π is a non-dimensional version of the physical quantity a obtained by dividing it by the adequate powers of the parameters a_i:

$$\Pi = \frac{a}{a_1^{\alpha_1} a_2^{\alpha_2} \cdots a_n^{\alpha_n}} \tag{3}$$

where $\alpha_1 \ldots \alpha_n$ are such that Π has no dimension. Several combinations might be possible. The Pi-theorem alone does not allow for the determination of the form of the function φ in Eq. (2). It however allows for a powerful generalization of experimental results. It is especially useful in fluid dynamics and heat and mass transfer. It leads for instance to the extrapolation of lab-scale experimental results to full-scale applications (Yarin 2012). Based on the Pi-theorem, the results of two experiments are indeed the same as long as the governing non-dimensional parameters are equal, even if the specific length- and/or time-scales of the dimensional parameters are different. This very important result is referred to as the similarity of two phenomena (Yarin 2012). For instance, it is therefore possible to predict the behavior of a plane based on lab-scale wind tunnel experiments, as long as the non-dimensional parameters that were found relevant are equal in both cases.

A well-known example of a non-dimensional parameter of interest in fluid dynamics is the Mach number. It compares the relative velocity of an object in a fluid (for example a plane in the atmosphere) to the speed of sound in this fluid:

$$Ma = \frac{v}{v_s} \tag{4}$$

As the Mach number is the ratio of two velocities (expressed for instance in m/s or in km/h, which doesn't matter as long as the units of both velocities are consistent), it has no dimension. If $Ma < 1$, the flow is subsonic. If $Ma > 1$, the flow is supersonic. It shows that the velocity of the plane alone doesn't rule the type of flow at stake: the Mach number does. The velocity of the plane needs to be compared to the local speed of sound, that varies with the temperature (and therefore with the altitude).

Another example of non-dimensional number is the Reynolds number Re. It compares the effects of viscous forces and inertial forces in a fluid, and it is used among others to determine whether a flow is laminar (no lateral mixing between fluid layers) or turbulent (lateral mixing through swirls). Re is defined as:

$$Re = \frac{d\upsilon}{\nu} \quad (5)$$

where υ is the velocity of the fluid, d is the characteristic length-scale of the problem (for instance the diameter of a pipe) and ν is the kinematic viscosity of the fluid. Re has no dimension. Two flows with the same Reynolds number therefore present the same characteristics, even if the fluid velocity, the fluid viscosity, and/or the diameter of the pipes are different. The relative importance of viscous forces compared to inertial forces rules the type of flow at stake, not the velocity, the viscosity, nor the diameter of the pipe alone.

Although these non-dimensional numbers are functions of the main physical parameters of a problem, they are usually expressed as constants (e.g., Re and not Re $[d, \upsilon, \nu]$). This is because that the parameters on which they are based are generally supposed to be constant for a given problem. We will use the same formalism for the extension of the Pi-theorem to team sports.

3 A relevant non-dimensional parameter

In this Section, we propose a governing non-dimensional number based on the main physical characteristics of football and hockey that can be correlated with the average number of goals scored per game in their various forms. This governing non-dimensional number will be a function of a limited number of non-dimensional variables derived from basic physical variables in the framework of the Pi-theorem. The combination of these non-dimensional variables will be determined a priori, based on an analysis of the physical problem at stake. The chosen approach is therefore mainly physical rather than statistical. The results presented here could also be obtained using purely statistical methods directly applied to the basic physical parameters. The main difference between those two approaches is the lack of explanatory power of purely statistical correlations (Falkenberg and Schiemann 2019), both in terms of physical phenomena and in terms of similarity of problems, the latter being brought to light through the use of the Pi-theorem.

The nine basic parameters taken into account in this study and their units following the International System of Units (SI) are given in Table 1. The ball velocity is the velocity of the ball when it is not possessed by a player, during a pass or a shot. The radius of action of a player is the distance from which a player can interact with another player. It is related to the surface of the field that can be considered as covered by a player and it will be used to quantify the player density on the field. In the framework of the Pi-theorem, the number of physical parameters n is therefore equal to nine. Figure 1 illustrates the nine parameters on a representative field.

Table 1: The physical parameters taken into account in this study and their units.

Symbol	Parameter	Units
t_g	Game duration	[s]
l_f	Field length	[m]
w_f	Field width	[m]
w_g	Goal width	[m]
h_g	Goal height	[m]
υ_p	Player velocity	[m/s]
υ_b	Ball velocity	[m/s]
n_p	Number of players	[-]
r_p	Player radius of action	[m]
N_g	Number of goals	[-]

The number of players and the number of goals scored have no dimension. As far as the other parameters are concerned, only lengths, velocities and durations are considered. The basic SI units at stake are therefore meters for the lengths, seconds for the durations, and their combination meters per second for the velocities. In the framework of the Pi-theorem, the number of independent dimensions k is therefore equal to two.

If the Pi-theorem can be extended to this problem, the average number of goals scored per game should be expressed as a function of maximum $(n - k) = 7$ non-dimensional parameters, see Section 2. Table 2 gives the seven non-dimensional parameters that can be derived from the original parameters by using the field length l_f and the player velocity υ_p as references, except for the height of the goal which is divided by the goal width to let the goal aspect ratio α_g appear. Similarly, the aspect ratio of the field

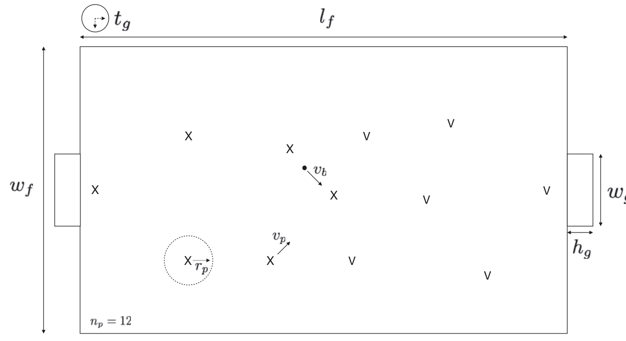

Figure 1: Illustration of the nine physical parameters of Table 1.

Table 2: The (n – k) non-dimensional parameters derived from the *n* basic parameters of Table 1.

Original parameter	Units	Derived non-dimensional parameter
l_f	[m]	-
v_p	[m/s]	-
t_g	[s]	$t_g^\star = \frac{t_g v_p}{l_f}$
w_f	[m]	$\alpha_f = \frac{w_f}{l_f}$
w_g	[m]	$w_g^\star = \frac{w_g}{l_f}$
h_g	[m]	$\alpha_g = \frac{h_g}{w_g}$
v_b	[m/s]	$v_b^\star = \frac{v_b}{v_p}$
r_p	[m]	$r_p^\star = \frac{r_p}{l_f}$
n_p	[-]	n_p
N_g	[-]	N_g

α_g is also defined as the ratio between the width and the length of the field. The number of players and the number of goals are obviously not modified, since they already are non-dimensional.

As explained in Section 2, the non-dimensional numbers defined in this work will be expressed as constants (without explicit reference to the parameters they depend on), as it is generally the case in physics.

A first, intuitive attempt to define a single non-dimensional number that could correlate with the number of goals scored per game would consist in comparing the duration of the game t_g to the time t_f needed for a player to cross the field: such a ratio could give a direct image of the number of scoring opportunities. The duration t_f can be defined as the ratio between the field length l_f and the representative player velocity v_p. The resulting non-dimensional number could therefore be expressed as follows:

$$A = \frac{t_g}{t_f} = \frac{t_g v_p}{l_f} = t_g^\star \tag{6}$$

which actually corresponds to the proposed non-dimensional version of the game duration t_g in Table 2, i.e., $t_g^\star$. The longer the game, the shorter the field and/or the faster the players, the higher their chances of scoring. As will be shown in Section 4, this simple approach proves unsuccessful when applied to the considered team sports: no correlation can be found between the non-dimensional number A and the average number of goals scored $\bar{N}_g$. Using the six other non-dimensional parameters of Table 2 will allow us to propose a more complex but successful model.

One of the reasons why A and $\bar{N}_g$ do not correlate is probably that a player does not need to cross the whole field in order to attempt to score. Instead, the notion of midfield must be introduced as the distance between the two scoring zones, where the players are close enough to the goals to have a good chance of scoring. The size of such scoring zones should at least be a function of the goal size and the ball velocity: the larger the goal and/or the faster the ball, the larger the distance from which an attempt to score can be successful.

A second reason why $\bar{N}_g$ is not correlated to A is probably that the average velocity on the field is not equal to the player velocity v_p, nor to the ball velocity v_b. It should be a function of those two values, as well as of the interactions between the players, that is in turn a function of the number of players on the field.

The non-dimensional number B proposed here therefore compares the duration of the game t_f to the time t_{mf} needed to cover the midfield length l_{mf}, based on a representative field velocity v_f:

$$B = \frac{t_g}{t_{mf}} = \frac{t_g v_f}{l_{mf}} \tag{7}$$

where both the representative midfield length l_{mf} and field velocity v_f still need to be defined using non-dimensional equations (i.e., where all terms are non-dimensional) based on the parameters of Table 2.

Considering s_{max} as the maximal distance from which an attempt to score becomes reasonable, the midfield length can be defined as the field length l_f minus two times $\frac{s_{max}}{2}$, taken as a representative distance from which actual scoring attempts are made:

$$l_{mf} = l_f - 2\frac{s_{max}}{2} = l_f - s_{max} \tag{8}$$

A representative value for s_{max} can be found by equaling the time needed for the ball to cover that distance at the velocity v_b and the time for the goalkeeper or a defender to cover half the representative size of the goal d_g at the velocity v_p:

$$\frac{s_{max}}{v_b} = \frac{d_g}{2v_p} \tag{9}$$

where d_g is defined as the diagonal of the goal:

$$d_g = \sqrt{h_g^2 + w_g^2} \tag{10}$$

It gives the following non-dimensional equation:

$$\frac{s_{max}}{\sqrt{l_g^2 + w_g^2}} = \frac{1}{2}\frac{v_b}{v_p} \tag{11}$$

The faster the ball, the slower the goalkeeper and/or the larger the goal, the longer the maximum distance from which it is reasonable to attempt to score (McGinnis 2013).

The length of the midfield can now be expressed only as a function of basic parameters:

$$l_{mf} = l_f - \frac{\upsilon_b}{\upsilon_p} \frac{\sqrt{h_g^2 + w_g^2}}{2} \tag{12}$$

or in a non-dimensional form:

$$\frac{l_{mf}}{l_f} = 1 - \frac{\upsilon_b}{\upsilon_p} \frac{\sqrt{h_g^2 + w_g^2}}{2\, l_f} \tag{13}$$

Taking into account the definition of the non-dimensional parameters of Table 2, it gives:

$$\frac{l_{mf}}{l_f} = 1 - \frac{1}{2}\, \upsilon_b^\star\, w_g^\star \sqrt{1 + \alpha_g^2} \tag{14}$$

This expression shows that, for a given field length, the players have a shorter distance to cover before they can attempt to score when the goals are larger and/or when the ball moves faster compared to the players.

The basis for a representative field velocity υ_f can be the average of υ_b and υ_p. In order to account for the interactions between the players, it should also be a function of the player density on the field, as well as the relative velocity of the players compared to the ball $\left(\frac{\upsilon_p}{\upsilon_b}\right)$: for a given number of players on the field, the faster the players compared to the ball, the easier it is to intercept a pass. The player density ρ_p can be assessed by comparing the total surface covered by the field players, based on their radius of action, to the surface of the field. The number of field players is the total number of players n_p minus the two goalkeepers. ρ_p is therefore defined as the following non-dimensional number:

$$\rho_p = \frac{(n_p - 2)\, \pi\, r_p^2}{w_f\, l_f} = (n_p - 2)\, \frac{\pi}{\alpha_f}\, r_p^{\star 2} \tag{15}$$

that is also only expressed as a function of non-dimensional parameters from Table 2.

The level of interaction on the field ε can be defined by combining the density of player and their relative velocity compared to the ball:

$$\varepsilon = \rho_p \frac{\upsilon_p}{\upsilon_b} = \frac{\rho_p}{\upsilon_b^\star} \tag{16}$$

It is not straightforward to determine a priori whether an increased number of players on the field tends to slow down or speed up the game. When the player density is low, additional players can speed up the game by offering more pass opportunities. However, additional players probably slow down the game when the player density reaches a certain threshold, as more and more opponents hinder the progression to the goal. This would be in line with the increased physiological demand observed when playing SSG during training (Halouani et al. 2014; Hammami et al. 2017; Sarmento et al. 2018). Therefore, the following expression is proposed for υ_f:

$$\upsilon_f = \frac{\upsilon_b + \upsilon_p}{2}\, \varepsilon\, (1 - \kappa\, \varepsilon) \tag{17}$$

where the non-dimensional parameter κ arbitrates between the positive and the negative impact of ε on υ_f: the quadratic form of the factor $\varepsilon\,(1 - \kappa\,\varepsilon)$ allows for an optimum field velocity, when the number of players is high enough to circulate the ball without too many interactions with the opponents. κ is the only adjustable parameter of the model. Its value will be determined later on to obtain the best possible correlation between the proposed non-dimensional parameter and the available data. In order to highlight its dependence on the value of κ, the proposed non-dimensional number will now be referred to as B_κ.

The non-dimensional version of Eq. (17) is:

$$\frac{\upsilon_f}{\upsilon_p} = (1 + \upsilon_b^\star)\, \frac{\varepsilon}{2}\, (1 - \kappa\, \varepsilon) \tag{18}$$

In summary, the proposed non-dimensional number B_κ can be calculated based on the seven non-dimensional parameters of Table 2 thanks to the following four non-dimensional equations:

$$B_\kappa = \frac{t_g}{t_{mf}} = \frac{t_g\, \upsilon_f}{l_{mf}} = t_g^\star\, \frac{l_f}{l_{mf}}\, \frac{\upsilon_f}{\upsilon_p} \tag{19}$$

$$\frac{l_{mf}}{l_f} = 1 - \frac{1}{2}\, \upsilon_b^\star\, w_g^\star \sqrt{1 + \alpha_g^2} \tag{20}$$

$$\frac{\upsilon_f}{\upsilon_p} = (1 + \upsilon_b^\star)\, \frac{\rho_p}{2\,\upsilon_b^\star} \left(1 - \kappa\, \frac{\rho_p}{\upsilon_b^\star}\right) \tag{21}$$

$$\rho_p = (n_p - 2)\, \frac{\pi}{\alpha_f}\, r_p^{\star 2} \tag{22}$$

Eq. (19) defines B_κ as the ratio between the duration of the game and the time needed to cross the midfield at the representative field velocity υ_f. Eq. (20) gives the relative length of the midfield as a function of the relative velocity of the ball and the relative size of the goals. Eq. (21) gives the relative field velocity as a function of the player density and the relative velocity of the players, using an adjustable parameter κ. Eq. (22) gives an expression for the player density ρ_p.

The remaining task is now to determine whether the defined non-dimensional number B_κ, which is a function of seven basic non-dimensional variables and one adjustable parameter, can actually correlate with the average number of goal scored per game.

4 Correlation between $\bar{N}_g$ and B

In this Section, we investigate the correlation between the non-dimensional number B_κ we have defined in Section 3 and the average number of goals scored per game during recent international competitions for the eight following team sports: Football (F), Futsal (Fs), Beach Soccer (BS), Field Hockey (FH), Indoor Hockey (InH), Hockey 5s (H5s), Ice Hockey (IcH), and Inline Hockey (IiH).

We seek a function f such that:

$$\bar{N}_g = f(B_\kappa) \quad (23)$$

where B_κ ultimately depends on the basic parameters of Table 2 following Eqs. (19)–(22). A priori, as B_κ compares the duration of the game to the order of magnitude of the time needed to cross the midfield, it is expected that $\bar{N}_g$ is a linear function of B_κ, with $\bar{N}_g = 0$ when $B_\kappa = 0$. We therefore seek a correlation of the following form:

$$\bar{N}_g = \alpha B_\kappa \quad (24)$$

The value of the parameter κ in Eq. (21) will be adjusted to obtain the best possible fit with the data retrieved from recent international competitions. The value of α will be derived from this optimal correlation.

4.1 Values of the parameters and statistical data

Table 3 gives the representative values of the basic parameters of Table 1 for the eight considered sports, as well as the average numbers of goals scored per game during the latest international competitions. A limitation of this work is that the number of goals scored per game can evolve in time, as reported by Leite et al. for Football, Beach Soccer, and Futsal (Leite and Barreira 2014). The reported variations however lie within the range of the standard errors on the mean that will be reported here, at least for the most recent competitions. Sports that are less mature, such as Hockey 5s and Inline Hockey could be subject to larger variations in the future, that might require an adjustment of the correlation that will be derived here.

Football presents the largest field and goal sizes, as well as the longest game duration (FIFA 2015a). The representative velocity of the players v_p is taken as half the average speed reached during the world record of running over 1 km: 3.79 m/s (or 13.6 km/h) (Guinness World Records 2020), which is slightly lower than what is generally reported as high-intensity running (>15 km/h) (Naser, Ali, and Macadam 2017). The same methodology is used for Ice

Table 3: Representative values of the basic parameters for the eight considered sports.

	l_f [m]	w_f [m]	w_g [m]	h_g [m]	v_p [m/s]	v_b [m/s]	t_g [s]	n_p [-]	r_p [m]	$\bar{N}_g$ [-]	Ref.
Football	105	69.5	7.32	2.44	3.79	24.7	5400	22	1	2.6	(FIFA 2015a, 2018; Guinness World Records 2020; Lees, Kershaw, and Moura 2005)
Futsal	40	22.5	3	2	3.79	24.7	2400	10	1	6.8	(FIFA 2014, FIFA 2016, Guinness Book of Records 2020; Lees, Kershaw, and Moura 2005)
Beach Soccer	36	27	5.5	2.2	3.19	23.7	2160	10	1	8.9	(Alcaraz et al. 2011; FIFA 2015b, 2019; Guinness World Records 2020; Lees, Kershaw, and Moura 2005; Potthast 2010)
Field Hockey	91.4	55	3.66	2.14	3.79	38.6	3600	22	1.5	4.4	(FIH 2018; Guinness World Records 2020; Sundar 2019; Wikipedia 2019a)
Indoor Hockey	44	22	3	2	3.79	38.6	2400	12	1.5	7.7	(FIH 2018; Guinness World Records 2020; Sundar 2019; Wikipedia, 2019b)
Hockey 5s	48	31.76	3.66	2.14	3.79	38.6	2160	10	1.5	8.6	(FIH 2019; Guinness World Records 2020; Sundar 2019; Youth Olympic Games 2014
Ice Hockey	52	27.5	1.83	1.22	7.56	44.7	3600	12	1.5	6.4	(Guinness World Records 2020; IIHF 2019a, 2019b; Sharp 2011)
Inline Hockey	42.4	25	1.7	1.05	6.12	44.7	2400	10	1.5	7.5	(Guinness Book of Records 2020; Sharp 2011; World Stake 2009a, 2009b)

Hockey and Inline Hockey. It results in representative velocities of 7.56 and 6.12 m/s for ice and inline roller skaters, respectively (Guinness World Records 2020). A representative velocity of the ball when it is kicked v_b is reported in (Lees, Kershaw, and Moura 2005): 24.7 m/s. The radius of action r_p is taken as 1 m. The same values of v_p, v_b, and r_p are used for Futsal. For Beach Soccer, the player velocity was reduced by 15.8% due to the effect of the sand running surface (Alcaraz et al. 2011). For the same reason, the ball velocity is also slightly reduced (–4%) (Potthast 2010). The average number of goals scored per game during the last FIFA men's World Cup (Russia 2018) is 2.6 (FIFA 2018). Futsal is an indoor variant of Football played on a smaller field by 10 players. The goal size and the duration of the game are also reduced, but less than the field size. The number of goals scored per game therefore increases: 6.8 in average during the 2016 FIFA Futsal World Cup in Colombia (FIFA 2016). The field size is also reduced in Beach Soccer, although it is larger than in Futsal. The goals are also larger, which results in an even higher number of goals scored per game: 8.9 during the 2019 FIFA Beach Soccer World Cup in Paraguay (FIFA 2019; Wikipedia 2019c).

Field Hockey is played on a pitch of approximately the same size as in Football by the same number of players (FIH 2018). Although the goals are smaller and the games are shorter (FIH 2018), Field Hockey players score more than Football players: 4.4 goals per game in average during the FIH 2018 Men's Hockey World Cup in India (Wikipedia 2019a). It can be explained by the fact that the hockey ball moves faster when it is hit: 38.6 m/s (Sundar 2019), versus 24.7 m/s for Football (Lees, Kershaw, and Moura 2005). The same value of v_b is considered for Indoor Hockey and Hockey 5s. As for Football and its variants, the representative player velocity v_p is taken as 3.79 m/s. The same value is considered for Indoor Hockey and Hockey 5s. As they play with a stick, the radius of action of the players increases: 1.5 m, versus 1 m for Football. This value is also used for all the other variants of Hockey. Indoor Hockey is played by 12 players on a smaller field, but the size of the goal is not significantly smaller (FIH 2015). Even though the games are also shorter (FIH 2015), more goals are therefore scored per game: 7.7 in average during the FIH 2018 Men's Indoor Hockey World Cup in Germany (Wikipedia 2019b). Hockey 5s is played by 10 players on a field slightly smaller than half a conventional Hockey field (FIH 2019). The goal size however remains the same. Despite the shorter game duration, this also leads to a significant increase of the number of goals scored per game: 8.6 in average during the Youth Olympic Games of Nanjin in 2014 (Youth Olympic Games 2014). Ice Hockey player moves much faster than Field Hockey players: 7.56 m/s, versus 3.79 m/s. Their puck also moves faster than a Hockey ball: 44.7 m/s versus 38.6 m/s (Sharp 2011). The same value is considered for an Inline Hockey puck. As the goals are not located at the extremities of the field, the parameter l_f is taken as the distance between the goals, and not as the total field length (IIHF 2019a). Even if the field sizes are comparable, and despite a longer game duration, Ice Hockey players score less than Indoor Hockey or Hockey 5s players: 6.4 goals per game in average during the IIHF 2019 Ice Hockey World Championship in Slovakia (IIHF 2019b). This lower scoring pace can be explained by the significantly smaller size of the goals (IIHF 2019a). The characteristics of Inline Hockey are very similar to those of Ice Hockey, excepted the shorter game duration and the lower speed of the players: 6.12 m/s. The shorter distance between the goals however seems to compensate this, as 7.5 goals per game were scored during the World Roller Game 2019 in Barcelona (World stake 2009b). As extremely high numbers of goals were observed for some games (up to 44), probably due to significant differences in the level of some teams, the games presenting goal differences higher than 10 were excluded from the data set.

4.2 Best-fit linear correlation

Figure 2 illustrates that lack of correlation between $\bar{N}_g$ and the non-dimensional number A defined in Eq. (6). The error bars illustrate the standard errors on the mean. For instance, the number A cannot be used to predict the higher number of goals scored in Field Hockey compared to Football. The longer game duration of Football is dominant, while in reality the higher velocity of the hockey ball

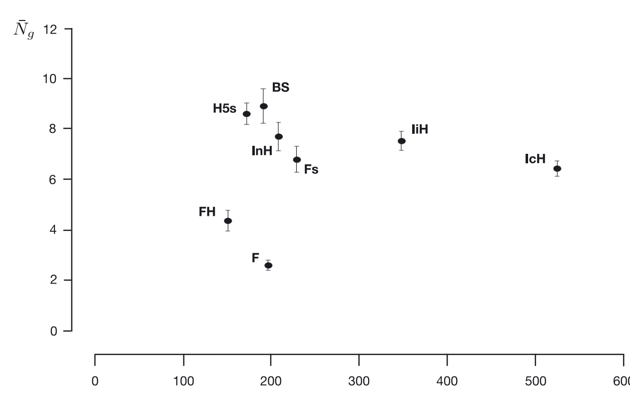

Figure 2: There is no correlation between the average number of goals per game and the non-dimensional number A. The error bars illustrate the standard errors on the mean.

probably plays a significant role in the higher scores reached. Figure 2 also shows that the A number of Ice Hockey is by far the highest, due to the high velocity of ice skaters. It fails to account for the high level of interaction on the field, and the smaller size of the goals.

When the goal size, the ball velocity and the number of players on the field are also taken into account following Eqs. (19)–(22), a strong correlation appears. Figure 3 shows that $\bar{N}_g$ is highly correlated to the non-dimensional number B_κ that is proposed here. The best linear correlation is found for $\kappa = 99$, and gives:

$$\bar{N}_g = 2.04\, B_\kappa \tag{25}$$

The corresponding R^2 value is 0.93. Such a good linear correlation between $\bar{N}_g$ and B_κ tends to demonstrate that the eight studied sports can indeed be considered similar, and that B_κ is a governing variable for the average number of goals scored per game. Therefore, it is also the parameter we use in order to predict the average outcome of any other modification brought to the basic characteristics of football or hockey, i.e., the outcome of any other possible variant.

5 Examples of ad hoc modifications

In this Section, the correlation of Eq. (25) is used to predict the effects of three ad hoc modifications of the basic parameters of Football, Hockey 5s and Indoor Hockey.

Football presents the longest game duration (90 min), but the lowest average number of goals scored per match (2.6), which can be deemed perfectible in terms of spectator experience. We propose here to investigate the effect of a reduction of the average Football field length on the expected average number of goals per game, all other parameters being kept constant. Figure 4 shows the resulting evolution of $\bar{N}_g$ as a function of l_f. Reducing the field length by 10 m already results in an increase of $\bar{N}_g$ by 0.55. The level of 3.5 (±0.6) goals per game is reached for a field length of 86 m (–19 m), and it is predicted that more than 4 (±0.5) goals per game will be scored if the field length is reduced to 81 m (–24 m).

Hockey 5s presents the highest scoring pace among the studied variants of hockey: 8.6 goals are scored in average during 36 min-long games. This can be explained by the fact that the goal size is identical to that of Field Hockey, while the field is significantly smaller. In order to reduce the number of game interruptions, it can be desired to decrease the number of scoring opportunities by reducing the size of the goals. Figure 5 shows the influence of a goal

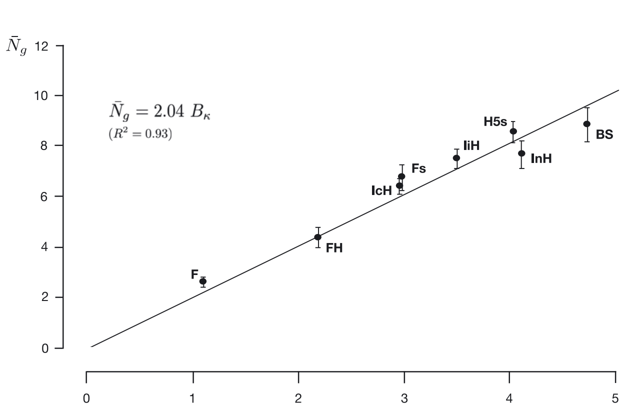

Figure 3: There is a very strong correlation between the average number of goals per game and the non-dimensional number B_κ, for $\kappa = 99$. The best-fit linear correlation is $\bar{N}_g = 2.04\, B_\kappa$ ($R^2 = 0.93$). The error bars illustrate the standard errors on the mean.

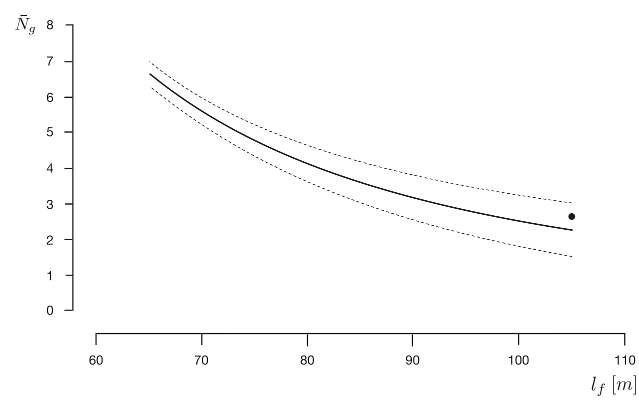

Figure 4: Impact of a field length reduction on the average number of goals scored per game in Football, all other parameters being kept constant. The dot indicates the current value. The dashed lines indicate the 90% confidence interval on the predicted mean.

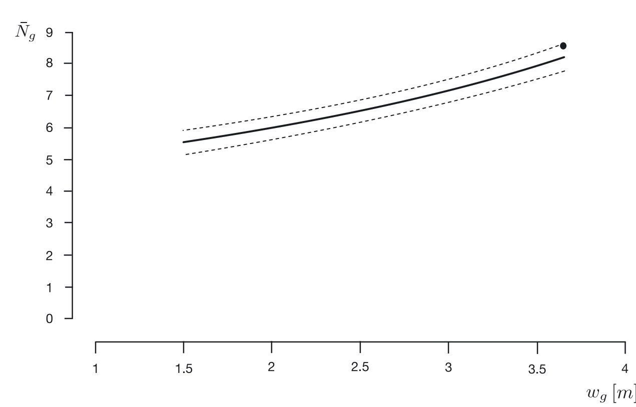

Figure 5: Impact of a goal width reduction on the average number of goals scored per game in Hockey 5s, all other parameters being kept constant (except the goal height, that varies proportionally with the goal width). The dot indicates the current value. The dashed lines indicate the 90% confidence interval on the predicted mean.

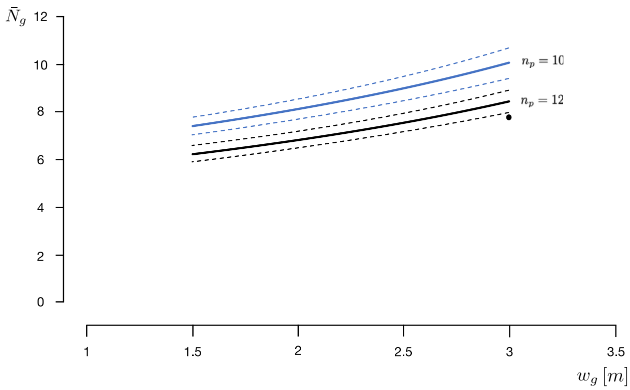

Figure 6: Combined impact of a lower number of players and a goal width reduction on the average number of goals scored per game in Indoor Hockey, all other parameters being kept constant (except the goal height, that varies proportionally with the goal width). The dot indicates the current values. The dashed lines indicate the 90% confidence interval on the predicted mean.

width reduction on $\bar{N}_g$, all other parameters being kept constant (except the goal height, that varies proportionally with the goal width). Reducing the goal size by 25% ($w_g = 2.75$ m), results in a decrease of the average number of goals to 6.9 (±0.3). A further reduction to 50% of the initial size ($w_g = 1.83$ m) leads to $\bar{N}_g = 5.9\,(\pm 0.4)$.

Among the studied sports, Indoor Hockey presents the highest player density. Decreasing the number of players on the field could decrease the number of interactions and improve the flow of the game. However, it would also increase the scoring pace. It could therefore be useful to decrease both the number of players and the goal size. Figure 6 shows the combined impact of the reduction of those two parameters, all other parameters being kept constant (except the goal height, that varies proportionally with the goal width). It is predicted that playing Indoor Hockey with 10 players while keeping the same goal size increases the number of goals scored per game up to 10 (±0.6). In order to keep $\bar{N}_g$ constant, the goal size needs to be reduced with 27% ($w_g = 2.19$ m).

6 Conclusions and future works

In this paper, we defined the non-dimensional number B_κ and we showed that it governs the number of goals scored per game during international competitions for eight variants of football and hockey. The B_κ number is based on the ratio between the game duration and the order of magnitude of the time needed to cross the midfield zone. It can be calculated using the basic characteristics of the considered sports: field size, goal size, representative player velocity, representative ball velocity, player density and game duration. The linear correlation between B_κ and $\bar{N}_g$ is excellent. In future works, the analysis carried out here should be extended to other sports, such as lacrosse, water polo, handball, basketball, American football, rugby, or ultimate to assess in which extent they can be considered as similar. The analysis could also be applied to female players, as well as other ages and levels.

We also used the derived correlation to predict the effects of ad hoc modifications of the basic parameters of the studied sports, such as the field size, the goal size, or the number of players. For instance, decreasing the length of football fields by 20 m would allow reaching more than 3.6 (±0.6) goals scored per game in average, versus 2.6 during the most recent men's World Cup. Reducing the Hockey 5s goal size with 25% results in a decrease of the average number of goals to 6.9 (±0.3), instead of 8.6. A further reduction to 50% of the initial size leads to $\bar{N}_g = 5.9\,(\pm 0.4)$. In Indoor Hockey, a combined reduction of the number of player to 10 and a goal size reduction of 27% would result in the same amount of goals scored per game, while improving the game flow. The effect of any other modification of the basic parameters of football, hockey and their variants can be studied using the proposed non-dimensional number and the derived correlation. The applied method can be used by sport federations to modify some characteristics of existing sports in order to adjust the scoring pace to a desired level or to investigate new variants of football and hockey.

Author contribution: All the authors have accepted responsibility for the entire content of this submitted manuscript and approved submission.
Research funding: None declared.
Conflict of interest statement: The authors declare no conflicts of interest regarding this article.

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