Evaluating recruitment strategies using fuzzy set theory in stochastic Manpower Planning

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Abstract

One aspect of Manpower Planning is the investigation of wastage and internal transitions for homogeneous groups of employees in a Manpower system. In the relevant literature, the attainability of a desired stock vector was studied under control by recruitment for time-discrete Markov models. These approaches allow choosing a proper recruitment strategy, resulting in an attainable vector most similar to a desired one. In this paper, this problem will be discussed under stochastic assumptions for attainability after one step. Based on fuzzy set theory, a procedure is introduced that allows evaluating and comparing recruitment strategies, resulting in the determination of a most preferable strategy.

Keywords: Manpower Planning, Stochastic models, Fuzzy set theory, Markov models, Attainability

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1. Introduction

Markov Chain theory is widely used in Manpower Planning for personnel mobility forecasting. Generally, the personnel is classified into \( k \) exclusive classes. The internal flows between those classes and the flows between the system and the external environment are investigated. The classes are determined by personnel characteristics (such as grades, salary, qualifications, experience) into which one wants to gain insights and/or that contribute to the homogeneity with respect to the transition probabilities [1]. The considered flows in Manpower Planning are recruitments, wastage and transitions between the homogeneous groups. Extensive literature exists about modeling the personnel system as Time-Homogeneous Markov systems (HMS) (e.g. [2,3,4]), Semi-Markov systems (e.g. [5,6]), Non Time-Homogeneous Markov systems (NHMS) (e.g. [7,8,9]) or Non Time-Homogeneous Semi-Markov models (e.g. [10,11]). The evolution of the personnel mobility can be studied continuous in time or time-discrete. In this paper, the discussion is restricted to a time-discrete point of view. The models allow studying the evolution of the number of people in every class, under a certain recruitment and/or promotion strategy. This enables organizations to investigate the impact of a recruitment and/or promotion policy on the future personnel stocks and if necessary, to adapt it in order to reach or approach the desired class sizes in the future. A lot of research is done on the so called attainability and maintainability problem of a desired personnel stock vector under control by recruitment and control by promotion [12,13,14,15,16,17,18,19,20]. This discussion in most research is restricted to an embedded Markov chain model, implying that the total number of individuals in the system is known (e.g. [2,12]). This assumption is convenient from a mathematical point of view to describe the set of attainable stock vectors (after \( t \) steps) and in the discussion whether a desirable stock vector is attainable or not under a certain recruitment policy. For example, for a system with constant total size the discussion can be based on Markov chain theory. For a personnel system with known total size, the recruitment policy is characterized by the distribution of new entrants in the system over the different classes, since
the number of recruitments in the system is a random variable, dependent on the known size of the system. Although Markov models are by nature stochastic, most research on the attainability and maintainability problem is done from a deterministic point of view: The actual numbers of individuals are random variables, but for analysis, they are replaced by their expected values [13,21]. Under deterministic assumptions, Guerry [22] examined the grade of attainability. By this concept, for a desired stock vector that is not attainable, based on a fuzzy set approach, a grade of attainability can be expressed. In the deterministic case, the proper recruitment strategy, resulting into the personnel structure most similar to the desired one, can be chosen. Several other optimization approaches identify the optimal choice within the set of acceptable strategies for the organization, based on a particular way of expressing the preference of a strategy [e.g. 23,24].

Relatively few researchers investigated the attainable stock vectors from a stochastically point of view for HMS (e.g. [13,14,25]). More recently, this investigation is done for NHMS (e.g. [12,21,26]). Up till now, in the stochastic case, only the probability of attaining a desired stock vector is investigated. A recruitment policy can be chosen such that this probability is maximized, but this approach does not take the desirability of the other possible outcomes into consideration. This problem is specific to stochastic models, since in deterministic models, there is no uncertainty with respect to the stock vector after $t$ steps.

In this paper the stated problem will be examined in HMS. The discussion is restricted to attainability after one step. A procedure will be introduced to evaluate the impact of different recruitment policies on the attainability of a desirable personnel structure, dealing with the uncertainty due to the variability of the class sizes under stochastic assumptions. This evaluation will take into account the probability as well as the desirability of all possible personnel structures in the future: The degree in which each of those stock vectors is desirable is based on its similarity to the desired personnel structure. In the evaluation of a recruitment
policy, not only the desirability of the most likely stock vector is considered, but also of all other possible outcomes for the stock vector and their probabilities to occur. This allows comparing recruitment strategies and determining the most preferable ones.

The approach in this paper has some main differences with previous work. Firstly, the assumption of a known system size is left behind. Consequently, the decision on the recruitment policy does not only embrace the distribution of new entrants in the system over the different classes but also the total number of entrants itself. The total number of recruitments in the system will not be a random variable anymore, but fully determined by the organization as a part of the strategy. The relaxation of this model assumption is very useful in practice: It might be that a stock vector not resulting in the same system size as the desired stock vector, is more preferable than stock vectors resulting in the same system size. Secondly, the procedure in this paper is an extension of the approach in the literature about stochastic NHMS. The motive of the introduction of NHMS was to get a generalization of the HMS [27]. This was successful for the deterministic case, but was not yet fully established in a stochastic context. Most often, the initial step in Manpower Planning is the estimation of the transition probabilities from observations of stock vectors and flows in the past [1,2,26]. Consequently, the uncertainty of the personnel structure is due to two sources of variation: The prediction error is caused by the use of the multinomial distribution of the flows while the estimation error is due to the uncertainty of the transition probabilities. Next to the prediction error, the uncertainty considered in previous work about stochastic NHMS is restricted to the introduction of a stochastic mechanism that at every time point determines the selection of a transition pattern out of a pool of possibilities [21].

In Section 2 of this paper, a brief review will be given of the stochastic HMS. Section 3 considers the uncertainty of the class sizes after a one step prediction, such that the probability of a future personnel structure under a certain recruitment policy can be studied. In Section 4,
for a given recruitment policy, the preference of a stock vector will be introduced based on fuzzy set theory. In Section 5, there will be introduced an evaluation score for a recruitment strategy. Finally, Section 6 provides an illustration of the procedure to compare recruitment policies in stochastic Time-Homogeneous Markov Manpower Planning systems.

2. HMS in Manpower Planning

Consider a manpower system with \( S = \{S_1, S_2, \ldots, S_k\} \) the set of all exclusive classes and the current stock vector at time \( t = 0 \) known and denoted by \( n(0) \). The objective is to study the class sizes at a discrete time scale \( t = 1, 2, \ldots \) . Denote by \( n[t,R(t)] \) the number of employees in class \( i \) at time \( t \) assuming that the number of recruitments in that class in period of time \([t-1,t)\) is given by \( R(t) \). Let us introduce the recruitment vector \( R(t) = (R_i(t)) \) and the stock vector \( n(t,R(t)) = (n_i[t,R(t)]) \). The transition matrix is \( P \) with elements \( p_{ij} \) representing the probability of an individual in class \( i \) to move towards class \( j \) after one period of time.

The evolution of the expected stock vector of the system is given by the difference equation:

\[
E\left[ n(t+1,R(t+1)) \right] = E\left[ n(t, R(t)) \right] P + R(t + 1) .
\]  

\[ (2.1) \]

The objective of this paper is to suggest a procedure to evaluate the recruitment policy \( R(t+1) \) after one step. For the current stock vector \( n(0) \), the interest is in predicting the stock vector at \( t = 1 \) under control by recruitment. For convenience, we will denote \( E\left[ n(t, R) \right] \) under recruitment policy \( R \) by \( \hat{n}_R(t) \). Under a certain recruitment policy \( R \), equation (2.1) can be rewritten as:

\[
\hat{n}_R(1) = n(0) \ P + R .
\]  

\[ (2.2) \]
The desired stock vector at $t = 1$ is denoted by $n^*$. From a deterministic point of view, $n^*$ is always attained for $R$ satisfying:

$$n^* = n(0) P + R.$$  \hspace{1cm} (2.3)

However, it might be possible that $R$ has one or more negative elements, meaning that redundancies are needed in those particular classes [2,13].

Since from a stochastic point of view, the transition probabilities are unknown, they have to be estimated based on a historical dataset of movements in the organization from $t = 0$ till $t = T$. Anderson and Goodman [28] developed a maximum likelihood estimator for the transition probabilities under Markov assumptions:

$$\hat{p}_{ij} = \frac{N_{ij}}{N_i} \quad \text{with} \quad N_{ij} = \sum_{t=0}^{T-1} n_{ij}(t) \quad \text{and} \quad N_i = \sum_{t=0}^{T-1} n_i(t).$$ \hspace{1cm} (2.4)

$n_{ij}(t)$ and $n_i(t)$ are observed numbers in the dataset, with $n_{ij}(t)$ the number of employees of class $i$ at time $t$ that are in class $j$ at time $t+1$. This estimator is a minimum variance unbiased estimator [29].

In literature on the attainability problem, usually a distinction is made between two different points of view. In the ‘optimal feedback control’ philosophy, the organization knows the future promotions and number of leavers in the system at the start of the time interval for which the recruitment policy should be determined. In the ‘optimal open-loop control’ philosophy, the decision on recruitment is made without having this information. This brings along that one should take into account that promotions and wastage are random variables [13,26]. The
suggested procedure in this paper corresponds with the open-loop philosophy. The transition matrix is not row stochastic; the wastage probabilities are given by:

\[ p_{i,k+1} = 1 - \sum_{j=1}^{k} p_{ij} \quad \text{for} \quad i = 1, \ldots, k. \]  \hspace{1cm} (2.5)

3. Variability of class sizes

In this section, the variability of the class sizes is investigated. When the estimated transition probabilities and the HMS are used for prediction, one clearly should take into account two sources of variation: Firstly, it is well known that the distribution of \( n(t) \) is multinomial. The prediction error is caused by using this multinomial distribution for extrapolation. Secondly, the estimation error is due to the uncertainty of the transition probabilities [30].

Prediction error

Having the multinomial distribution approximated by the multivariate normal distribution, Sales [31] has pointed out that the number of employees at \( t \) is approximately normally distributed with mean \( \hat{n}_r(t) \) and variance-covariance matrix \( \Sigma_p(t) \). Bartholomew [32] has computed the elements of \( \Sigma_p(t) \) for HMS with stochastic recruitment and for multi-step prediction. The formula is easily simplified for the model with deterministic recruitment and one step prediction; the elements of \( \Sigma_p = \Sigma_p(1) \) are:

\[ \text{Cov}\{n_j(1), n_i(1)\} = \sum_i (\delta_{ji} p_{ij} - p_{ij} p_{ii}) \cdot n_i(0) \quad \text{with} \; \delta_{ij} \; \text{the Kronecker’s delta.} \]  \hspace{1cm} (3.1)
Estimation error

The transition probabilities used in the multinomial distribution have to be estimated by (2.4). Let us introduce the notations \( p_i = (p_{i1}, p_{i2}, \ldots, p_{ik}) \) and \( \hat{p}_i = (\hat{p}_{i1}, \hat{p}_{i2}, \ldots, \hat{p}_{ik}) \). It is well known [28,30], that asymptotically, for every \( i = 1, 2, \ldots, k \) the variable \( \sqrt{N_i} (\hat{p}_i - p_i) \) has a \( k \)-variate normal distribution with zero mean vector and variance-covariance matrix with elements:

\[
\sigma_{ji}^2 = p_{ij} (\delta_{ji} - p_{ii}) \quad j = 1, 2, \ldots, k \quad \text{with } \delta_{ji} \text{ the Kronecker's delta.} \quad (3.2)
\]

Using the distribution of \( \sqrt{N_i} (\hat{p}_i - p_i) \), we know that \( n_i(0) \hat{p}_i \) is asymptotically \( k \)-variate normal distributed with expected value \( n_i(0) p_i \) and variance covariance matrix \( \sum_i(i) \) with elements:

\[
\sigma_{ji}^2 = (\delta_{ji} p_{ij} - p_{ij} p_{jj}) \frac{n_i^2(0)}{N_i} \quad \text{with } \delta_{ji} \text{ the Kronecker's delta.} \quad (3.3)
\]

Using the fact that the sum of \( k \) jointly normal random variables is also normal, leads us to the conclusion that \( \hat{n}_R(1) = n(0) \hat{P} + R \) has a \( k \)-variate normal distribution with mean \( n(0)P + R \). Hence, \( n(1) - \hat{n}_R(1) \) follows approximately a \( k \)-variate normal distribution with zero mean vector.

We assume that \( n(0) \) is independent from the data set on which the maximum likelihood estimator is based. Hence, \( n(0) \) is independent of \( \hat{P} \) and the variance-covariance matrix \( \sum(1) \) of \( n(1) - \hat{n}_R(1) \) becomes:
\[ \Sigma(1) = Var[n(1) - \hat{n}_R(1)] \\
= Var[n(1)] + Var[\hat{n}_R(1)] \\
= \Sigma_p + \Sigma_e \]  
\[ (3.4) \]

with \( \Sigma_e = \sum_i \Sigma_e(i) \).  
\[ (3.5) \]

By replacing the elements of the variance-covariance matrix by its ML estimates resulting in the matrix \( \hat{\Sigma}(1) \), we get:

\[ n(1) - \hat{n}_R(1) \sim N_k[\hat{0}, \hat{\Sigma}(1)] \]  
\[ (3.6) \]

4. **Stochastic attainability and the preference of a stock vector**

To enable to compare recruitment strategies, the goal is to express the degree in which a recruitment strategy is preferable or not. This degree depends on the properties of the possible realizations for the stock vector under that strategy considered. Therefore in this section in first instant, for a given recruitment policy, a measure for the preference of a stock vector will be introduced. Based on this discussion, in Section 5, there will be introduced an evaluation score for a recruitment strategy, in order to express the degree in which a strategy is preferable.

The preference of a stock vector is related to the attainability of a desired stock vector \( n^* \). In that discussion under stochastic assumptions, one has to deal with two aspects:

- The uncertainty due to the stochastic assumptions (as pointed out in Section 3). For a starting stock vector \( n(0) \), a particular stock vector \( n(1) \) will be realized with a certain probability.
- The degree in which the stock vector \( n(1) \) is similar to a desired vector \( n^* \).

In what follows there will be introduced on the one hand a measure to express the degree in which a stock vector \( n \) will be realized (after one step) and on the other hand a measure to
express the degree in which $n$ is desirable. There will be assigned a higher preference to a vector $n$ in case it is more likely that $n$ will be realized and $n$ is more similar to a desired stock vector $n^*$. A fuzzy rule will be introduced to express the idea that for a vector $n$, the degree of preference is determined by both these aspects.

**The degree in which a stock vector is expected to be realized**

The vector $\hat{n}_R(1) = n(0)\hat{P} + R$ is the expected realization for the stock vector after one step for the considered recruitment strategy $R$. To quantify the fact that not all the vectors will be attained with equal expectancy, a membership function $R(n, R)$ will be introduced based on the probability density function $f$ of $n(1) \sim N_k [\hat{n}_R(1), \hat{\Sigma}(1)]$ and having for $n = \hat{n}_R(1)$ the value equal to 1. The degree in which a stock vector $n$ will be realized, for the considered recruitment strategy $R$ will be expressed by:

$$R(n, R) = \frac{f(n)}{f(\hat{n})} \quad \text{with} \quad f(n) = \frac{1}{2\pi^{\frac{k}{2}} \sqrt{|\hat{\Sigma}|}} \exp\left[ -\frac{1}{2} (n - \hat{n})' \hat{\Sigma}^{-1} (n - \hat{n}) \right]$$ \hspace{1cm} (4.1)

**The degree in which a stock vector is desirable**

A recruitment strategy is better in case in a greater extent the evolution of the stock vector is in the direction of a desired goal. Therefore the evaluation of a recruitment strategy is related to the discussion of attainability. In order to attain a desired stock vector it is important to deal with the problem of controlling parameters. In previous work attainability is studied in a deterministic approach, among under other conditions, under control by promotion and control by recruitment [2,12,13,14,15,16,17,18,19,20].

Within the set of strategies that are acceptable for the company, preferences for the strategy in achieving the goal(s) can be taken into account. By doing this it can be reflected whether some
way of achieving the goal(s) may be preferable to others. It can be for example that a personnel strategy is more preferred as the corresponding cost is less [23,24,33] or as the speed of converging is faster [34] or as the strategy is at the same time efficient in reaching the goal(s) and deviates as little as possible from a preferred strategy [35]. A goal programming approach was introduced in [36] to determine optimal manpower policies for the Canadian forces by minimizing the weighted sum of deviations of the constraints. And in [37] a model has been developed with the objectives of minimizing the manpower system costs and determining optimal recruitment strategies by a dynamic programming approach. By considering a fuzzy set approach, a preferred strategy can be selected based on the concept of the grade of attainability, i.e. the degree of similarity between attainable stock vectors and the desired vector [22].

To express the degree in which a vector is desirable, for each grade $i$ the fuzzy set $\{(n_i, D_i(n_i))\}$ in $\Theta$ is introduced. The membership function $D_i : \Theta \rightarrow [0,1]$ quantifies for each possible grade size $n_i$ the degree of similarity with the $i$-th component of the desirable structure $n^*$. In particular $D_i(n_i^*) = 1$ for the desired class size $n_i^*$ and $D_i(n_i) = 0$ in case $n_i$ is totally unacceptable as size for the $i$-th grade. The degree of similarity of $n_i$ to $n_i^*$ is determined by the discrepancy $n_i^* - n_i$. The membership function can vary with the grade $i$, this will be for example the case when one wants to reflect that a discrepancy in size for a particular grade is less or more acceptable. Depending on the characteristics of the manpower system, alternative membership functions are suitable [38]. One can consider for example a triangular fuzzy number to express that for grade $i$ no grade size less than a lower bound $a_i$ and no grade size greater than an upper bound $b_i$ can be accepted:
\[
D_i(n_i) = \begin{cases} 
0 & \text{for } n_i < a_i \text{ or } n_i > b_i \\
\frac{n_i - a_i}{n_i^* - a_i} & \text{for } a_i \leq n_i \leq n_i^* \\
\frac{n_i - b_i}{n_i^* - b_i} & \text{for } n_i^* \leq n_i \leq b_i 
\end{cases} \quad (4.2)
\]

Since a vector \( n \) is desirable in case all its components are, the degree in which \( n \) is desirable can be expressed by the membership function based on the minimum-operator:

\[
D(n) = \min_i D_i(n_i) \quad (4.3)
\]

The greater the value of \( D(n) \), the more the vector \( n \) is preferred.

**The preference of a stock vector**

Since the preference of vector \( n \) is determined by the degree \( D(n) \) of similarity with the desired vector \( n^* \) and by taking into account that the greater the value of \( R(n, R) \) the more likely \( n \) will be realized, the preference of the vector \( n \) can be quantified by:

\[
P(n, R) = \min \{ R(n, R), D(n) \} \quad (4.4)
\]

By considering the minimum-operator, both the similarity to the desired structure as the degree in which a stock vector is expected to be realized are required to be high to result in a high preference of the vector.
5. Evaluating a recruitment strategy

In a manpower system controlled by recruitment the objective is to determine a recruitment policy resulting in realizations for the stocks with a high level of preference. Therefore in comparing recruitment strategies a recruitment strategy is said to be highly preferred if there does exist a vector \( n \) with high preference \( P(n, R) \) for the considered recruitment vector \( R \). A preferable recruitment strategy is therefore characterized by a great value for \( \max_{n} P(n, R) \).

The evaluation of a recruitment strategy \( R \) is therefore based on the evaluation score:

\[
S(R) = \max_{n} P(n, R) \tag{5.1}
\]

In the stochastic approach, as introduced in this paper, the evaluation of a recruitment strategy is based on the preference of all stock vectors (and not only of the expected stock vector). For a recruitment strategy \( R \), the evaluation score \( S(R) \) equals the maximum value of \( P(n, R) \) over all stock vectors \( n \), with \( P(n, R) = \min\{R(n, R), D(n)\} \). Therefore \( S(R) \) equals the preference \( P(n', R) \) of a stock vector \( n' \) for which the degree \( D(n') \) in which it is desirable is strictly greater than zero. Consequently, in case \( D(n) \) is expressed by a triangular fuzzy number, the value of the evaluation score \( S(R) \) can be found by comparing the preference \( P(n, R) \) for all stock vectors \( n \) satisfying \( a_i \leq n_i \leq b_i \) or \( n_i^* \leq n_i \leq b_i \), for all \( i \).

In comparing the preference of recruitment strategies, the strategy \( \tilde{R} \) will said to be more preferred than the strategy \( \bar{R} \) if \( S(\tilde{R}) > S(\bar{R}) \). And a strategy \( \tilde{R} \) is a most preferable recruitment strategy in case \( \max_{R} S(R) = S(\tilde{R}) \).
6. Illustration

To illustrate the way of evaluating a recruitment strategy in a stochastic approach based on fuzzy set theory, as introduced in this paper, we analyze an organization for which the historical dataset of stocks and flows is given in Table 1. The number of homogeneous groups in the manpower system under study is 3. The notation $W_i(t)$ refers to the wastage out of grade $i$, i.e. the number of members of grade $i$ that are leaving the manpower system, during the considered period of time $[t,t+1)$.

<insert Table 1: Historical dataset>

Based on these historical data and according to (2.4), the ML estimates of the transition probabilities are:

$$
\hat{P} = \begin{pmatrix}
0.791 & 0.102 & 0.056 \\
0.062 & 0.739 & 0.102 \\
0.049 & 0.049 & 0.802
\end{pmatrix}.
$$

Given that the present stock vector is $n(0) = (200 \quad 275 \quad 225)$, independent from the dataset in Table 1, the variance-covariance matrix with respect to the prediction error (according to (3.1)) is

$$
\Sigma_p = \begin{pmatrix}
59.46623 & -29.1788 & -19.4071 \\
-29.1788 & 81.80579 & -30.6419 \\
-19.4071 & -30.6419 & 71.37213
\end{pmatrix}
$$

and the variance-covariance matrix with respect to the estimation error (according to (3.5)) is

$$
\Sigma_e = \begin{pmatrix}
6.68354 & -3.30418 & -2.29003 \\
-3.30418 & 10.9993 & -4.478 \\
-2.29003 & -4.478 & 9.852953
\end{pmatrix},
$$

resulting in the estimated variance-covariance matrix (according to (3.4))
\[
\hat{\Sigma}(1) = \begin{pmatrix}
66.15 & -32.48 & -21.70 \\
-32.48 & 92.80 & -35.12 \\
-21.70 & -35.12 & 81.23
\end{pmatrix}.
\]

For next year the desired stock vector is \( n^* = (200 \ 260 \ 230) \) and a discrepancy of three or more employees from the desired level is for the company not considerable. Therefore for each grade \( i \), the degree in which a stock \( n_i \) is desirable, is expressed by the triangular fuzzy number with lower bound \( a_i = 3 \) and upper bound \( b_i = 3 \).

In Table 2 there are gathered values of the evaluation scores for some recruitment strategies.

<insert Table 2: Evaluation scores for some recruitment strategies>

In comparing the recruitment strategies \( \tilde{R} = (14 \ 27 \ 10) \) and \( \bar{R} = (13 \ 24 \ 9) \), it can be found that the expected stock vectors after one step are respectively \( \hat{n}_{\tilde{R}} = n(0)P + \tilde{R} = (200 \ 262 \ 230) \) and \( \hat{n}_{\bar{R}} = (199 \ 259 \ 229) \).

In a deterministic approach, the evaluation of a recruitment strategy \( R \) is restricted to the discussion of the preference \( P(\hat{n}_R, R) \) of the expected stock vector \( \hat{n}_R \). In fact \( S(R) = P(\hat{n}_R, \tilde{R}) \) and moreover \( S(R) = P(\hat{n}_R, \bar{R}) = D(\hat{n}_R) \). Since the preferences of \( \hat{n}_{\tilde{R}} \) and \( \hat{n}_{\bar{R}} \) for respectively the recruitment strategies \( \tilde{R} \) and \( \bar{R} \), are equal to \( \frac{1}{3} \) and \( \frac{2}{3} \), in a deterministic approach the conclusion would be that the recruitment strategy \( \bar{R} \) is more preferred than the recruitment strategy \( \tilde{R} \).
For each recruitment strategy $R$, the value of the evaluation score $S(R)$ can be found by comparing the preference $P(n, R)$ for all stock vectors $n$ with coordinates that have a discrepancy less than three from the desired ones (since for the manpower system under consideration, a stock that differs more than three from the desired one is not considerable). This procedure results in an evaluation score for the recruitment strategy $\tilde{R}$ and $\bar{R}$ respectively equal to $S(\tilde{R}) \approx 0.9734$ and $S(\bar{R}) \approx 0.9135$. For both these recruitment strategies $\tilde{R}$ and $\bar{R}$ the maximum value of the preference is reached for the desirable structure $n^* = (200 \ 260 \ 230)$. In fact $S(\tilde{R}) = P(n^*, \tilde{R})$ and $S(\bar{R}) = P(n^*, \bar{R})$. The reasons for that is that the structure $n^*$ is, obviously, desirable with degree 1 and that moreover for the recruitment strategies $\tilde{R}$ and $\bar{R}$ this structure $n^*$ has a great value for the degree in which it is expected to be realized.

In the stochastic approach, the discussion on the evaluation of a strategy is based on more information than in the deterministic approach, and results in the conclusion that the recruitment strategy $\tilde{R}$ is more preferred than the strategy $\bar{R}$, in contrast with the conclusion drawn in the deterministic approach.

As presented in Table 2 the recruitment strategy $R = (14 \ 28 \ 10)$ results in the expected stock vector $\hat{n} = (200 \ 263 \ 230)$ with degree of desirability equal to 0, and therefore also with degree of preference equal to 0. In the deterministic approach the evaluation of the recruitment strategy $R = (14 \ 28 \ 10)$ is only based on the degree of desirability of the structure $\hat{n}$ that is, for the manpower system under study, not considerable since $D(\hat{n}) = 0$. In contrast with this finding, in the stochastic approach the evaluation score for this recruitment strategy $R$ has a rather high value, namely $S(R) = P(n^*, R) = 0.9311$. 
For the recruitment strategy \( R = \begin{pmatrix} 20 & 19 & 13 \end{pmatrix} \) it can be remarked that for the structure \( n = \begin{pmatrix} 201 & 259 & 230 \end{pmatrix} \) holds that \( S(R) = P(n, R) = D(n) = 0.6667 \). Meaning that the evaluation score \( S(R) \) corresponds with the degree in which the structure \( n \) is desirable.

For the recruitment strategy \( R = \begin{pmatrix} 8 & 36 & 7 \end{pmatrix} \) the evaluation score is determined by the degree in which the structure \( n = \begin{pmatrix} 199 & 261 & 230 \end{pmatrix} \) is expected to be reached since \( S(R) = P(n, R) = R(n, R) = 0.5953 \).

Finally it can be pointed out that for the recruitment strategy \( R = \begin{pmatrix} 14 & 25 & 10 \end{pmatrix} \) the expected stock vector equals the desirable stock vector \( n^* \). Consequently \( S(R) = P(n^*, R) = 1 \).

The discussion of the manpower system as case study emphasizes different aspects in the evaluation of recruitment strategies in a stochastic approach based on fuzzy set theory.

### 7. Conclusion

The main contribution of this paper is the presentation of a method for evaluating recruitment policies in stochastic Manpower Planning based on Time-Homogeneous Markov theory. In this way, a most preferable recruitment strategy can be chosen. Therefore, for every possible recruitment strategy, the degree of desirability is not only considered for the most likely future stock vector, but also for all other possible outcomes. Moreover, the method considers the degree in which all possible outcomes are expected to be realized. In case that the recruitment policy, resulting in the expected stock vector equal to the desired one, is one of the recruitment policies taken into consideration, the fuzzy set approach will prefer this recruitment policy above all other alternatives. This might give the impression that the deterministic approach of
the (by nature) stochastic attainability problem in previous work, leads to the same results as the method described in this paper. Nevertheless, the proposed methodology allows evaluating and expressing preferences for recruitment strategies in case this optimal recruitment policy is not one of the alternative strategies. The illustration in Section 6 shows that, by considering the stochastic aspects of the models, the recruitment policy resulting in the most likely stock vector closest to the desired one, might not always be the most preferred recruitment strategy.

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Table 1: Historical dataset

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<th></th>
<th>$n_1(t)$</th>
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Table 2: Evaluation scores for some recruitment strategies

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